

19UEC904 – CONTROL ENGINEERING

UNIT III - FREQUENCY RESPONSE ANALYSIS

•Closed loop frequency response- Performance specification in frequency domain- Frequency response of standard second order system - Bode Plot, Polar Plot, Nyquist Plot - Design of compensators using Bode plots: Cascade lag compensation - Cascade lag-lead compensation

Topics to be covered

- **Bode Plot**
- **Polar Plot**
- **Nyquist Plot**
- **Frequency Domain specifications**
- **Series-parallel Compensators**
- **Lead, Lag, and Lead Lag Compensators**

KEY PROBLEM OF CONTROL: STABILITY AND SYSTEM PERFORMANCE

QUESTION: WHY FREQUENCY RESPONSE

ANALYSIS:

1. Weakness of root locus method relies on the existence of open-loop transfer function
2. Weakness of time-domain analysis method is that time response is very difficult to obtain
 - ❖ **Computational complex**
 - ❖ **Difficult for higher order system**
 - ❖ **Difficult to partition into main parts**
 - ❖ **Not easy to show the effects by graphical method**

Frequency Response

Three advantages:

- * Frequency response (mathematical modeling) can be obtained directly by experimental approaches.
- * Easy to analyze effects of the system with sinusoidal signals
- * Convenient to measure system sensitivity to noise and parameter variations

Frequency domain analysis is a kind of (indirect method) engineering method. It studies the system based on frequency response which is also a kind of mathematical model.

- *Advantages*

- *Stability of closed loop system can be estimated*
- *Transfer function of complicated systems can be determined experimentally by frequency tests*
- *Effects of noise disturbance and parameter variations are relatively easy to visualize.*
- *Analysis can be extended to certain nonlinear control systems.*

Summary

- *Every measurement system require analysis of its features or performance to work as a system.*
- *Time domain analysis gives the behaviour of the signal over time. This allows predictions and regression models for the signal.*
- *Frequency Analysis is much easier. Some equations can't be solved in time domain while they can be solved easily in frequency domain.*

Frequency Response - Introduction

The response of a system for the sinusoidal input is called sinusoidal response. The ratio of sinusoidal response and sinusoidal input is called *sinusoidal transfer function* of the system and in general, it is denoted by $T(j\omega)$. The sinusoidal transfer function is the frequency domain representation of the system, and so it is also called *frequency domain transfer function*.

The frequency response of a system directly tells us the relative magnitude and phase of a system's output sinusoid if the system input is a sinusoid.

What about output frequency?

If the plant's transfer function is $g(s)$, the open-loop frequency response is $g(j\omega)$.

In time response analysis, we have considered the use of standard test inputs, such as step functions and ramps.

However, we will now consider the steady-state response of a system to a sinusoidal input test signal.

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However, we will now consider the steady-state response of a system to a sinusoidal input test signal.

One advantage of the frequency response approach is that we can use data derived from measurements on the physical system without deriving its mathematical model.

3.2 FREQUENCY DOMAIN SPECIFICATIONS

The performance and characteristics of a system in frequency domain are measured in terms of frequency domain specifications. The requirements of a system to be designed are usually specified in terms of these specifications.

The frequency domain specifications are,

1. Resonant peak, M_r
2. Resonant Frequency, ω_r
3. Bandwidth, ω_b
4. Cut-off rate
5. Gain margin, K_g
6. Phase margin, γ

Resonant Peak (M_r)

The maximum value of the magnitude of the closed loop transfer function

Resonant Frequency (ω_r)

The frequency at which the resonant peak occurs is called resonant frequency, ω_r . This is related to the frequency of oscillation in the step response and thus it is indicative of the speed of transient response.

Bandwidth (ω_b)

The Bandwidth is the range of frequencies for which normalized gain of the system is more than -3 db. The frequency at which the gain is -3 db is called cut-off frequency.

Cut-off Rate

The slope of the log-magnitude curve near the cut off frequency is called cut-off rate. The cut -off rate indicates the ability of the system to distinguish the signal from noise.

Gain Margin , K_g

The gain margin, K_g is defined as the value of gain, to be added to system, in order to bring the system to the verge of instability.

The gain margin, K_g is given by the reciprocal of the magnitude of open loop transfer function at phase cross over frequency. The frequency at which the phase of open loop transfer function is 180° is called the phase cross-over frequency, ω_{pc} .

$$\text{Gain Margin, } K_g = \frac{1}{|G(j\omega_{pc})|} \quad \dots(3.4)$$

Phase Margin (γ)

The phase margin γ , is defined as the additional phase lag to be added at the gain cross over frequency in order to bring the system to the verge of instability. The gain cross over frequency ω_{gc} is the frequency at which the magnitude of the open loop transfer function is unity (or it is the frequency at which the db magnitude is zero).

The phase margin γ , is obtained by adding 180° to the phase angle ϕ of the open loop transfer function at the gain cross over frequency

$$\text{Phase margin, } \gamma = 180^\circ + \phi_{gc}, \quad \dots(3.6)$$

$$\text{where, } \phi_{gc} = \angle G(j\omega_{gc})$$

Frequency Response plots

Frequency response analysis of control systems can be carried either analytically or graphically. The various graphical techniques available for frequency response analysis are,

1. Bode plot
2. Polar plot (or Nyquist plot)
3. Nichols plot
4. M and N circles
5. Nichols chart

The frequency response plots are used to determine the frequency domain specifications, to study the stability of the systems and to adjust the gain of the system to satisfy the desired specifications.

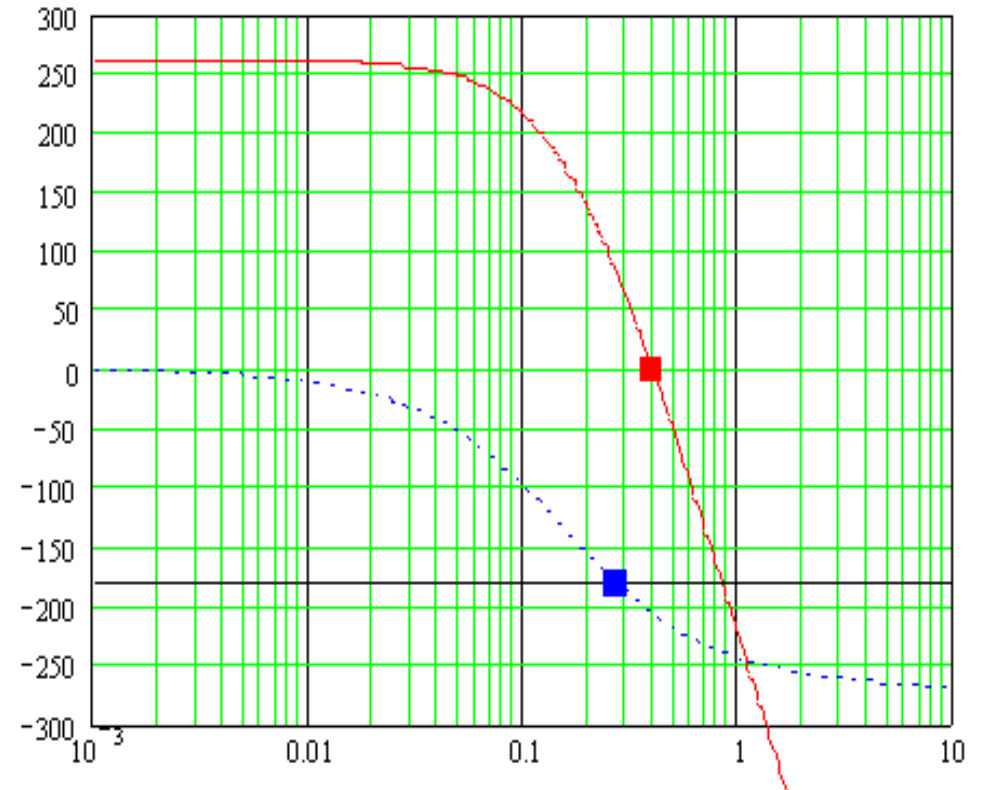
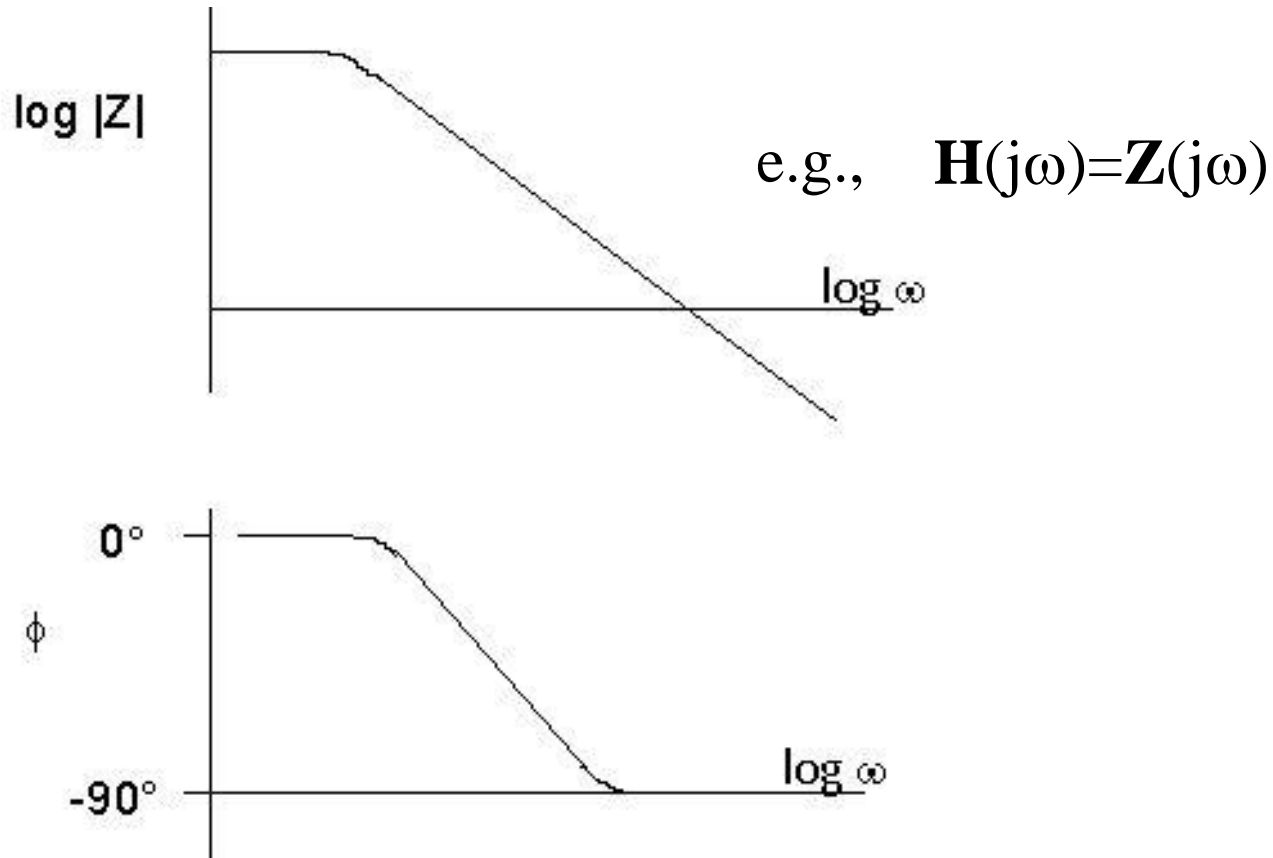
What is Bode Plot?

- Bode Plot is a frequency response plot of the sinusoidal transfer function .
- It consist of two graphs
- Magnitude versus log frequency ($\log w$)
- Phase angle versus log frequency ($\log w$)

It can be drawn for both open loop and closed loop system.

- ▶ The transfer function can be separated into magnitude and phase angle information

$$H(j\omega) = |H(j\omega)| \angle \Phi(j\omega)$$



Consider the open loop transfer function, $G(s) = \frac{K(1+sT_1)}{s(1+sT_2)(1+sT_3)}$

$$G(j\omega) = \frac{K(1+j\omega T_1)}{j\omega(1+j\omega T_2)(1+j\omega T_3)}$$
$$= \frac{K \angle 0^\circ \sqrt{1+\omega^2 T_1^2} \angle \tan^{-1} \omega T_1}{\omega \angle 90^\circ \sqrt{1+\omega^2 T_2^2} \angle \tan^{-1} \omega T_2 \sqrt{1+\omega^2 T_3^2} \angle \tan^{-1} \omega T_3}$$

The magnitude of $G(j\omega) = |G(j\omega)| = \frac{K \sqrt{1+\omega^2 T_1^2}}{\omega \sqrt{1+\omega^2 T_2^2} \sqrt{1+\omega^2 T_3^2}}$

The phase angle of the $G(j\omega) = \angle G(j\omega) = \tan^{-1} \omega T_1 - 90^\circ - \tan^{-1} \omega T_2 - \tan^{-1} \omega T_3$

The basic factors that very frequently occur in a typical transfer function $G(j\omega)$ are,

1. Constant gain, K
2. Integral factor, $\frac{K}{j\omega}$ or $\frac{K}{(j\omega)^n}$
3. Derivative factor, $K \times j\omega$ or $K \times (j\omega)^n$
4. First order factor in denominator, $\frac{1}{1+j\omega T}$ or $\frac{1}{(1+j\omega T)^m}$
5. First order factor in numerator, $(1+j\omega T)$ or $(1+j\omega T)^m$
6. Quadratic factor in denominator, $\left[\frac{1}{1+2\zeta(j\omega/\omega_n) + (j\omega/\omega_n)^2} \right]$
7. Quadratic factor in numerator, $\left[1+2\zeta\left(\frac{j\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2 \right]$

Example #1

Sketch Bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

SOLUTION

The sinusoidal transfer function $G(j\omega)$ is obtained by replacing s by $j\omega$ in the given s -domain transfer function.

$$\therefore G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

$$\text{Let } K=1, \therefore G(j\omega) = \frac{(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$$

$$\text{Let } K=1, \therefore G(j\omega) = \frac{(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \frac{1}{0.2} = 5 \text{ rad/sec}$ and $\omega_{c2} = \frac{1}{0.02} = 50 \text{ rad/sec}$

The various terms of $G(j\omega)$ are listed in Table-1 in the increasing order of their corner frequency.

The basic factors that very frequently occur in a typical transfer function $G(j\omega)$ are,

1. Constant gain, K
2. Integral factor, $\frac{K}{j\omega}$ or $\frac{K}{(j\omega)^n}$
3. Derivative factor, $K \times j\omega$ or $K \times (j\omega)^n$
4. First order factor in denominator, $\frac{1}{1+j\omega T}$ or $\frac{1}{(1+j\omega T)^m}$
5. First order factor in numerator, $(1+j\omega T)$ or $(1+j\omega T)^m$
6. Quadratic factor in denominator, $\left[\frac{1}{1+2\zeta(j\omega/\omega_n)+(j\omega/\omega_n)^2} \right]$
7. Quadratic factor in numerator, $\left[1+2\zeta\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2 \right]$

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$(j\omega)^2$ $\frac{1}{1+j0.2\omega}$ $\frac{1}{1+j0.02\omega}$	<p style="text-align: center;">-</p> $\omega_{c1} = \frac{1}{0.2} = 5$ $\omega_{c2} = \frac{1}{0.02} = 50$		$40 - 20 = 20$ $20 - 20 = 0$

Choose a low frequency ω_1 such that $\omega_1 < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_1 = 0.5$ rad/sec and $\omega_h = 100$ rad/sec.

Let, $A = |G(j\omega)|$ in db.

Let us calculate A at $\omega_1, \omega_{c1}, \omega_{c2}$ and ω_h .

$$\text{At } \omega = \omega_1, \quad A = 20 \log |(j\omega)^2| = 20 \log (\omega)^2 = 20 \log (0.5)^2 = -12 \text{ db}$$

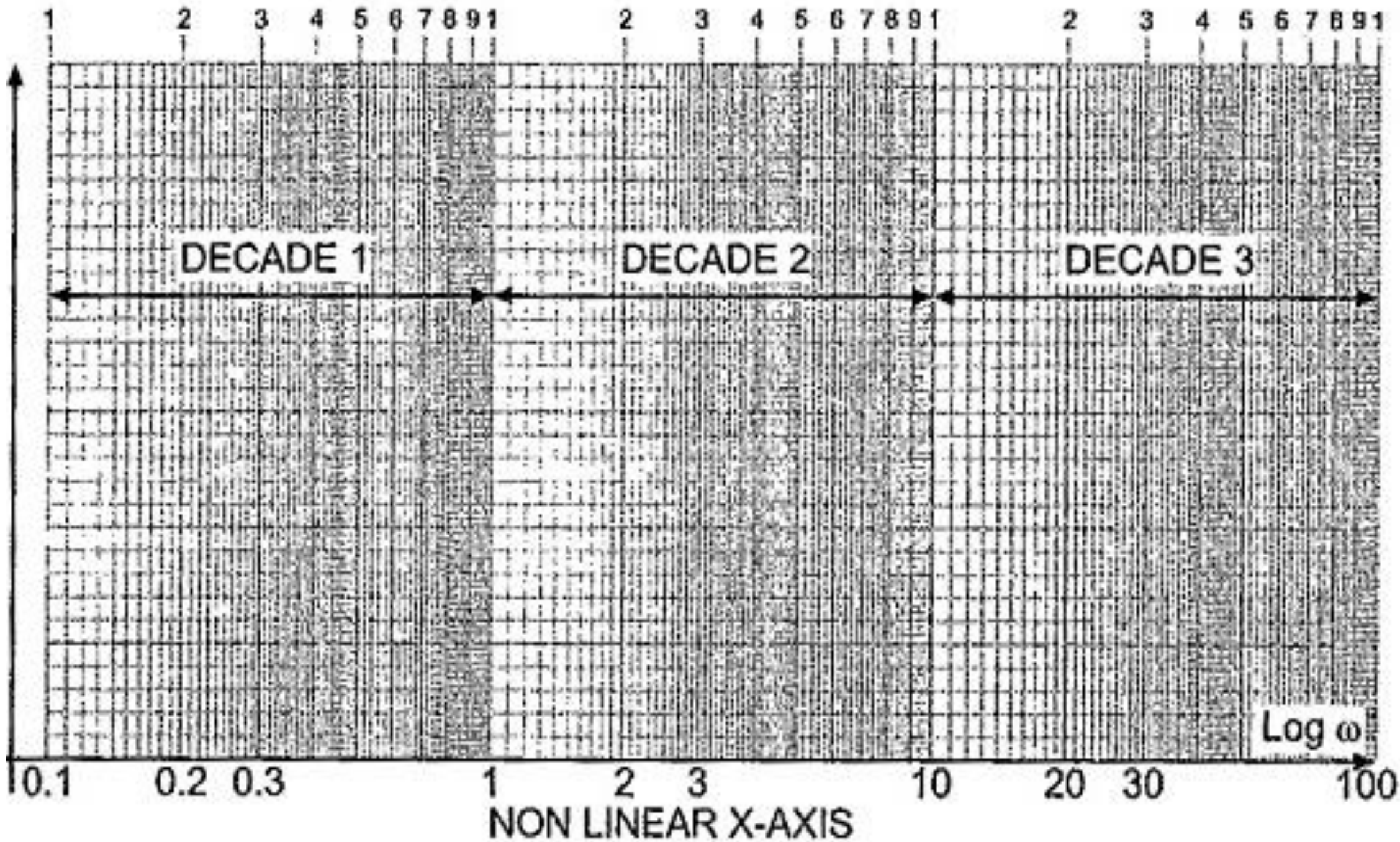
$$\text{At } \omega = \omega_{c1}, \quad A = 20 \log |(j\omega)^2| = 20 \log (\omega)^2 = 20 \log (5)^2 = 28 \text{ db}$$

$$\text{At } \omega = \omega_{c2}, \quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A_{(\text{at } \omega = \omega_{c1})} = 20 \times \log \frac{50}{5} + 28 = 48 \text{ db}$$

$$\text{At } \omega = \omega_h, \quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{(\text{at } \omega = \omega_{c2})} = 0 \times \log \frac{100}{50} + 48 = 48 \text{ db}$$

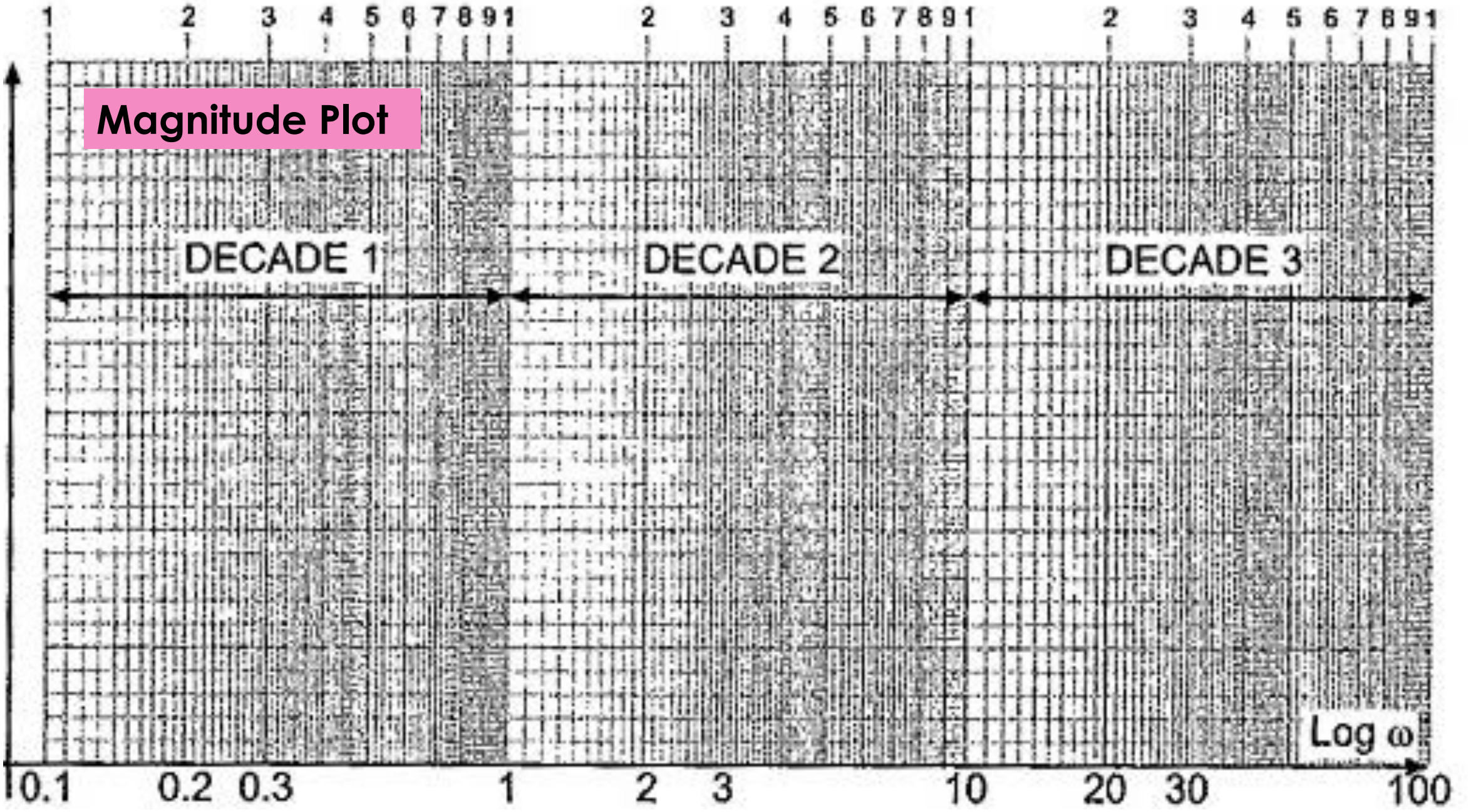
Let the points a, b, c and d be the points corresponding to frequencies ω_1 , ω_{c1} , ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1 unit = 10db on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on, logarithmic scales in x-axis. Fix the points a, b, c and d on the graph. Join the points by straight lines and mark the slope on the respective region.

Linear Y Axis



Magnitude Plot

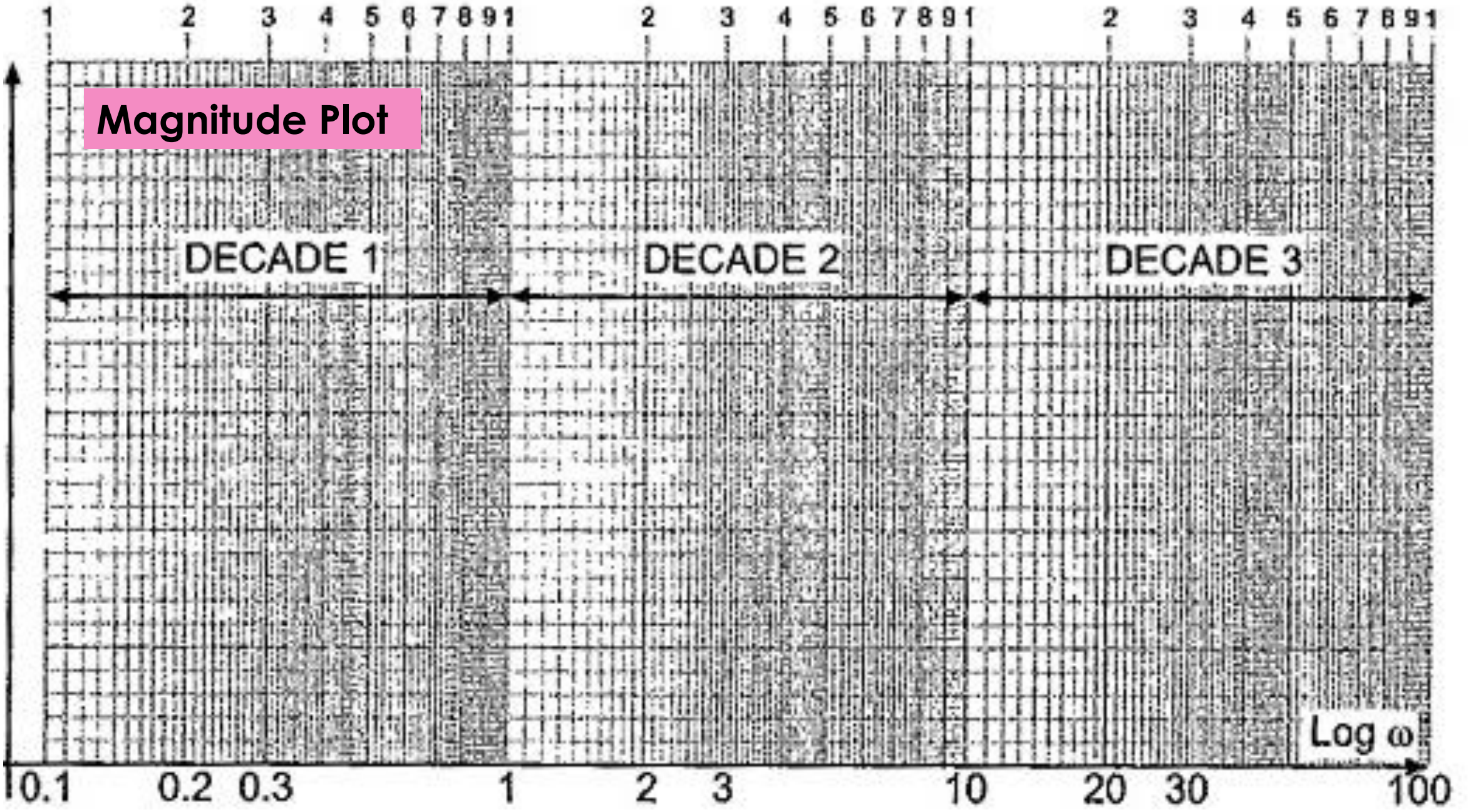
A in db



W in rad/sec

Magnitude Plot

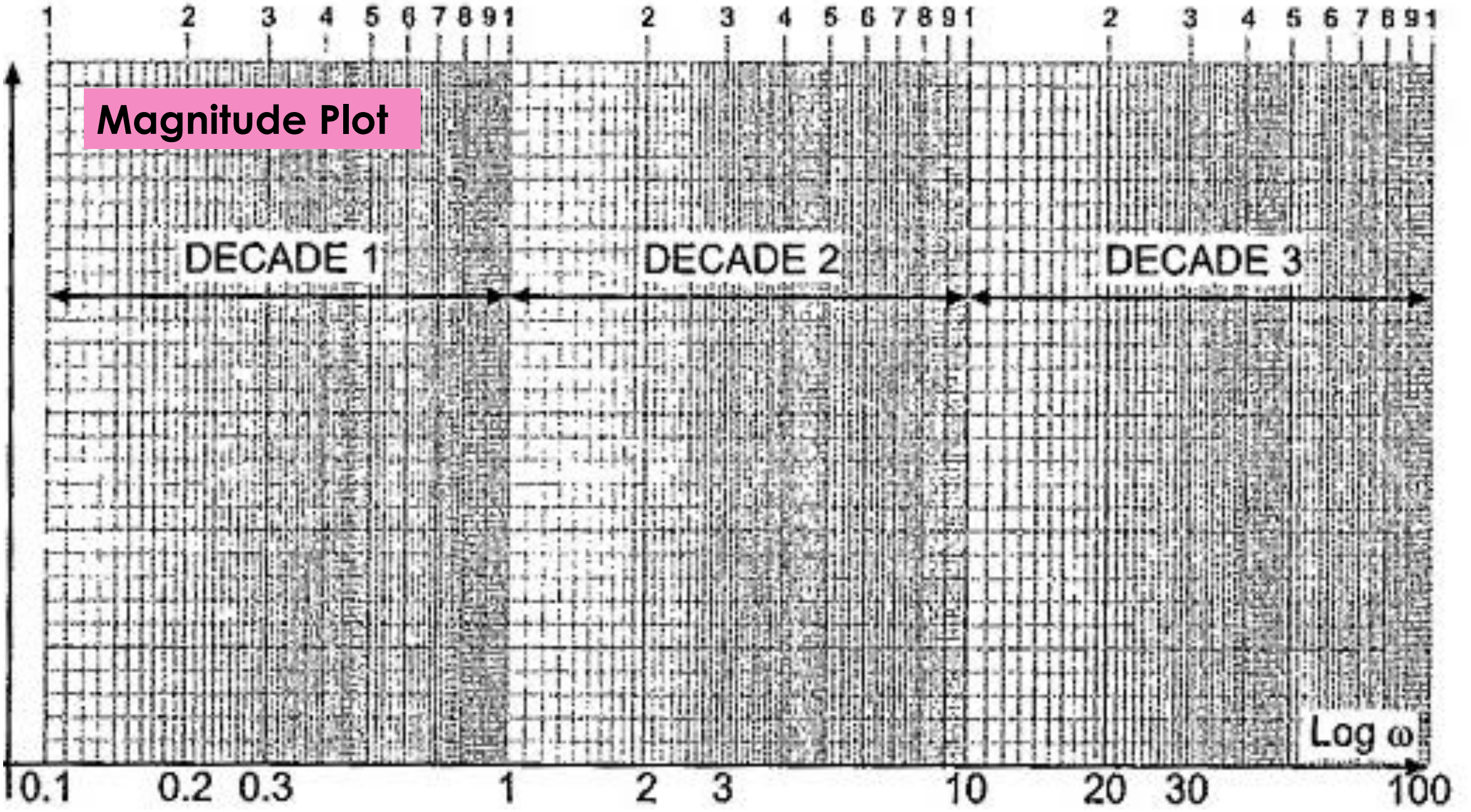
A in db



W in rad/sec

Magnitude Plot

A in db



W in rad/sec

PHASE PLOT

The phase angle of $G(j\omega)$ as a function of ω is given by,

$$\phi = \angle G(j\omega) = 180^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.02\omega$$

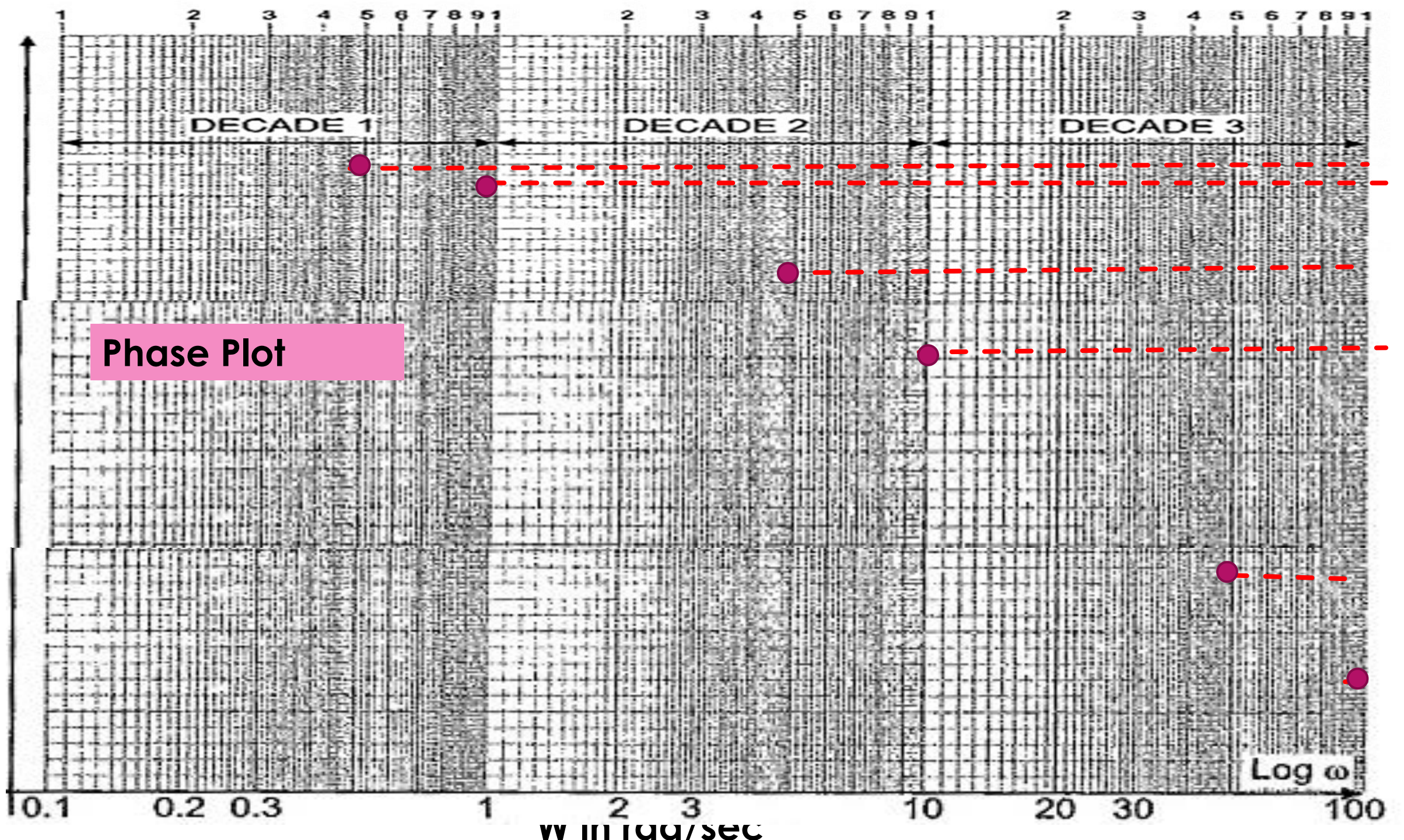
The phase angle of $G(j\omega)$ are calculated for various values of ω and listed in table-2.

$$\phi = \angle G(j\omega) = 180^\circ - \tan^{-1} 0.2\omega - \tan^{-1} 0.02\omega$$

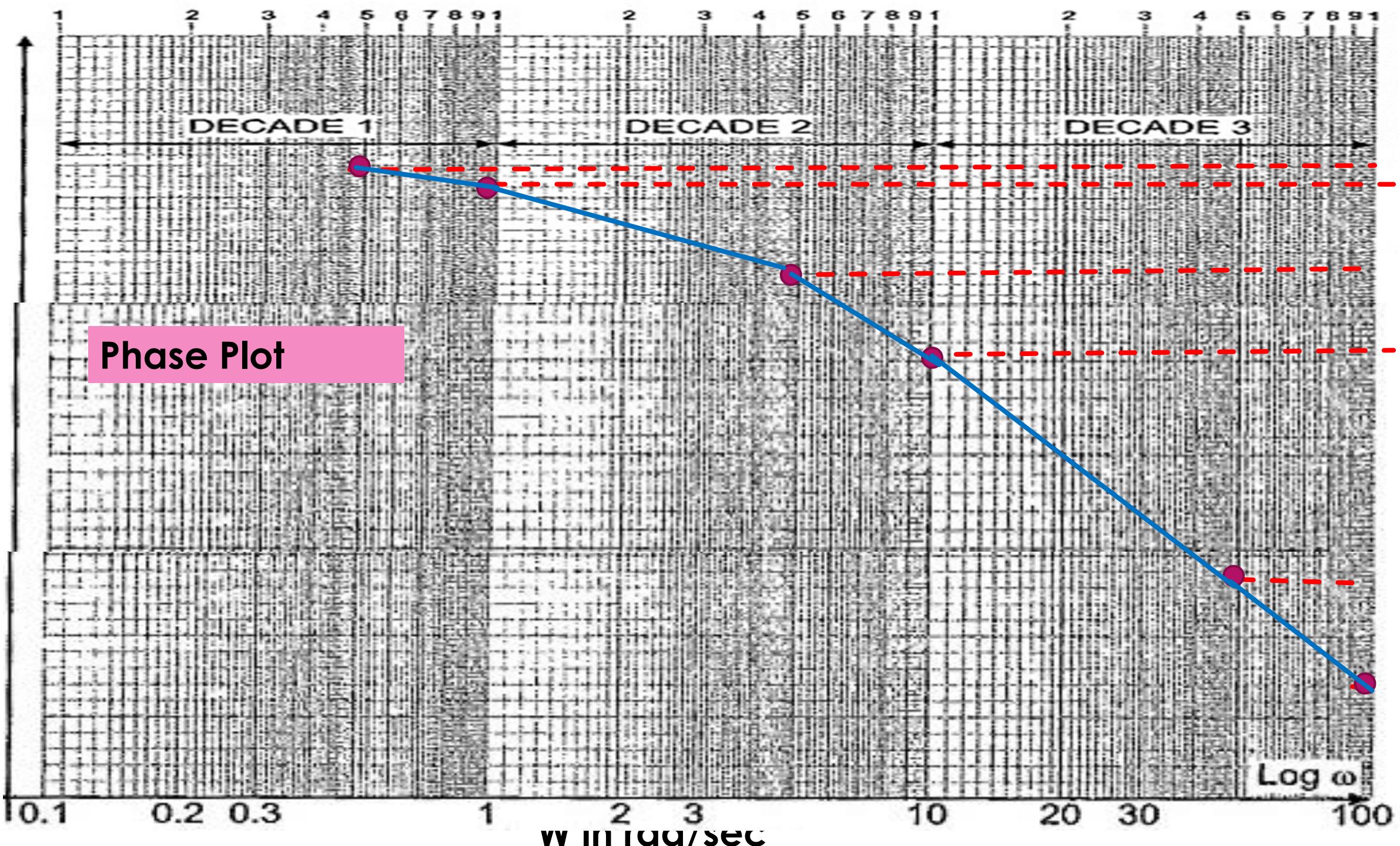
TABLE-2

ω rad/sec	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg	Point in phase plot
0.5	5.7	0.6	$173.7 \approx 174$	e
1	11.3	1.1	$167.6 \approx 168$	f
5	45	5.7	$129.3 \approx 130$	g
10	63.4	11.3	$105.3 \approx 106$	h
50	84.3	45	$50.7 \approx 50$	i
100	87.1	63.4	$29.5 \approx 30$	j

A in db



A in db



On the same semilog graph sheet choose a scale of 1 unit = 20°, on the y-axis on the right side of semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve.

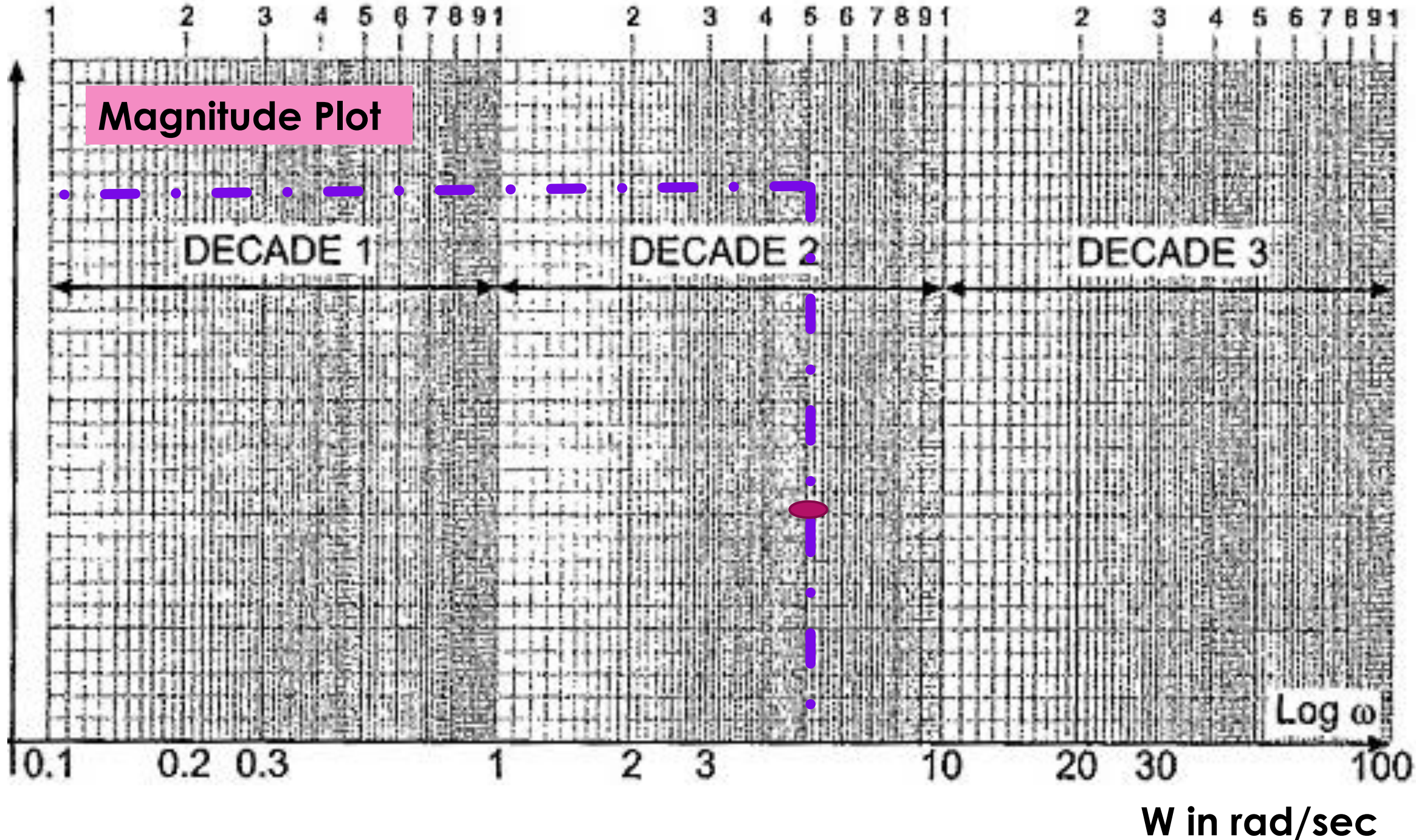
CALCULATION OF K

$$\therefore 20 \log K = -28 \text{ db}$$

$$\log K = \frac{-28}{20}; K = 10^{-\left(\frac{28}{20}\right)} = 0.0398$$

The magnitude plot with $K = 1$ and 0.0398 and the phase plot are shown

A in db

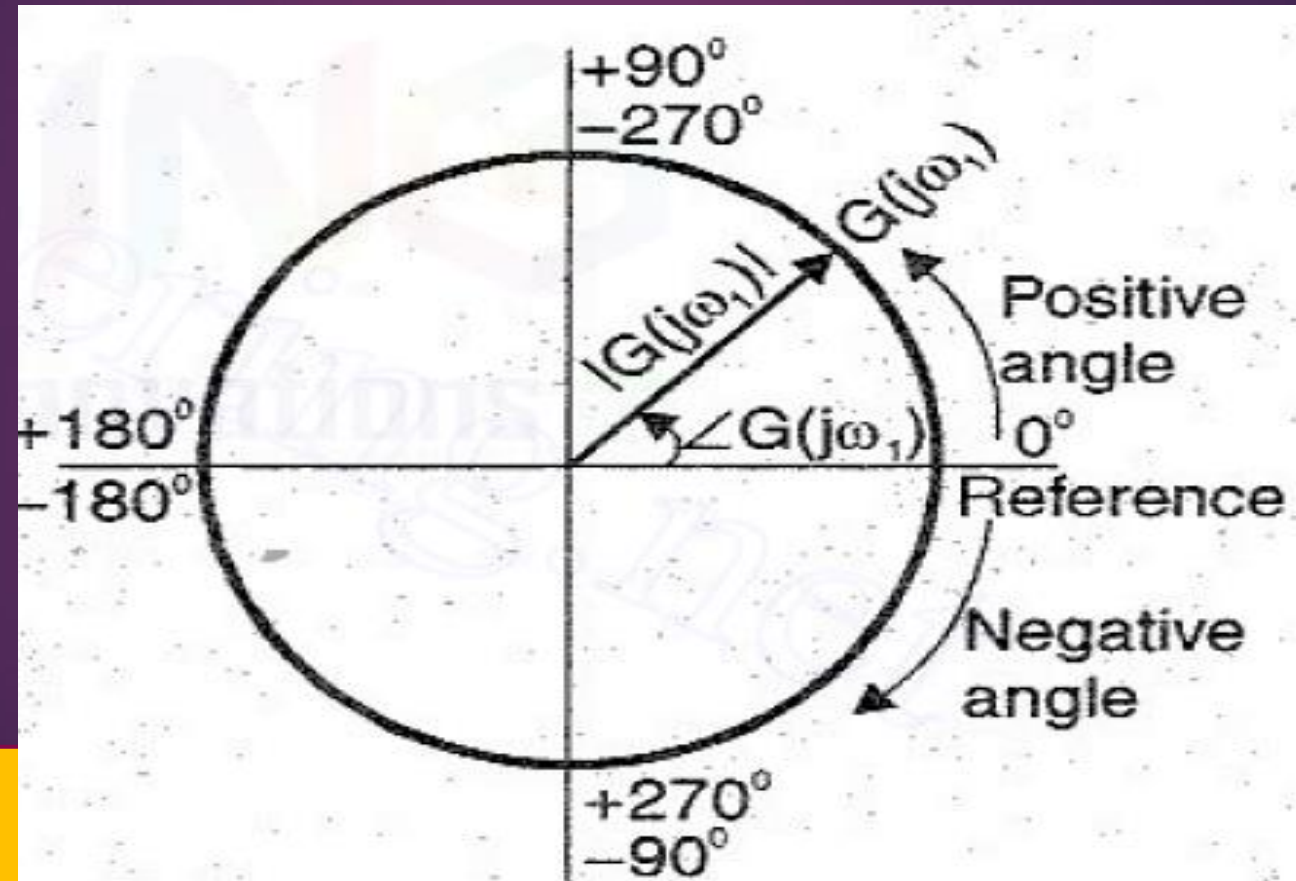


POLAR PLOT

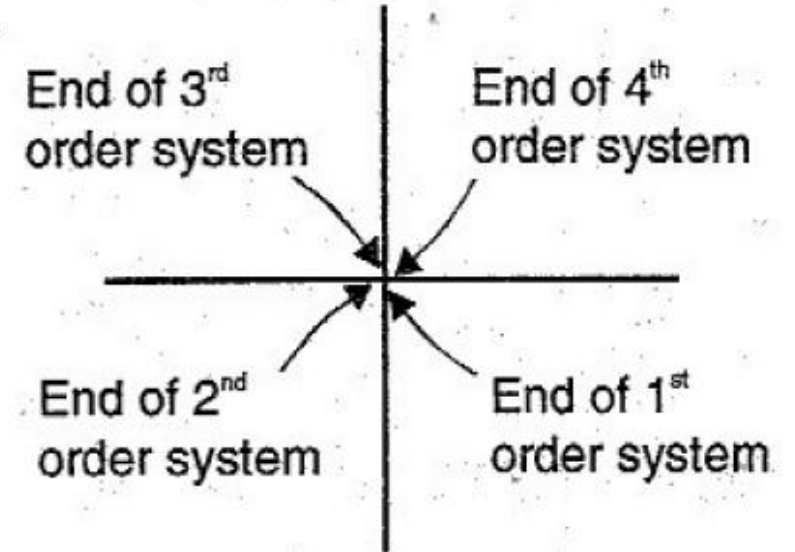
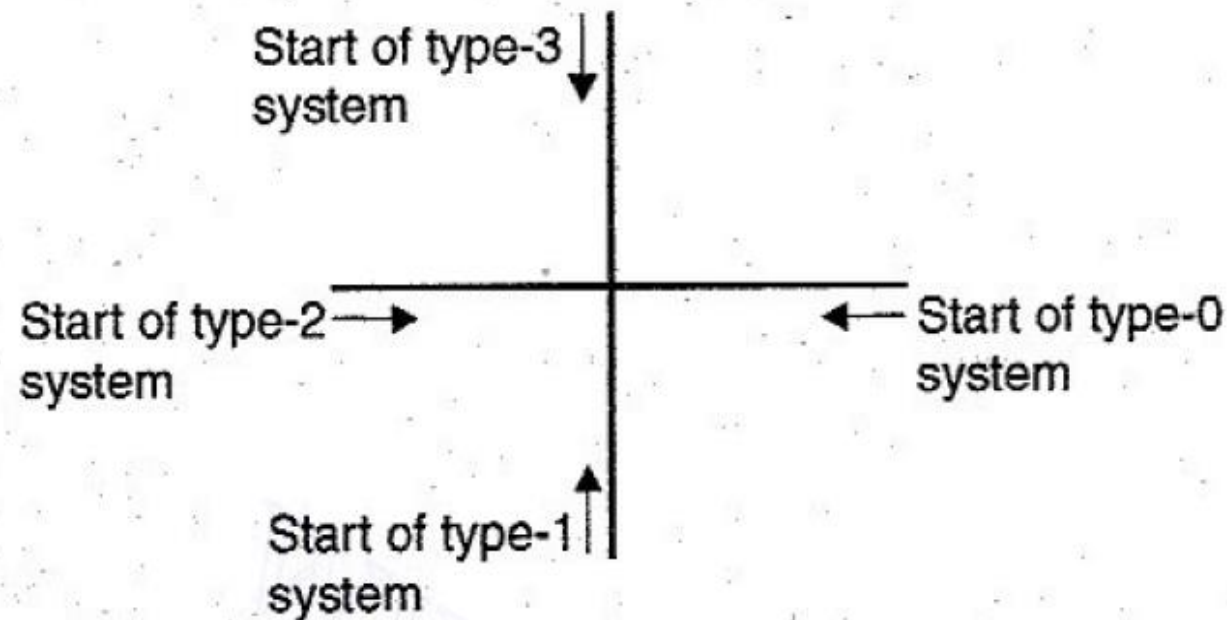
The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity. Thus the polar plot is the locus of vectors $|G(j\omega)| \angle G(j\omega)$ as ω is varied from zero to infinity. The polar plot is also called *Nyquist plot*.

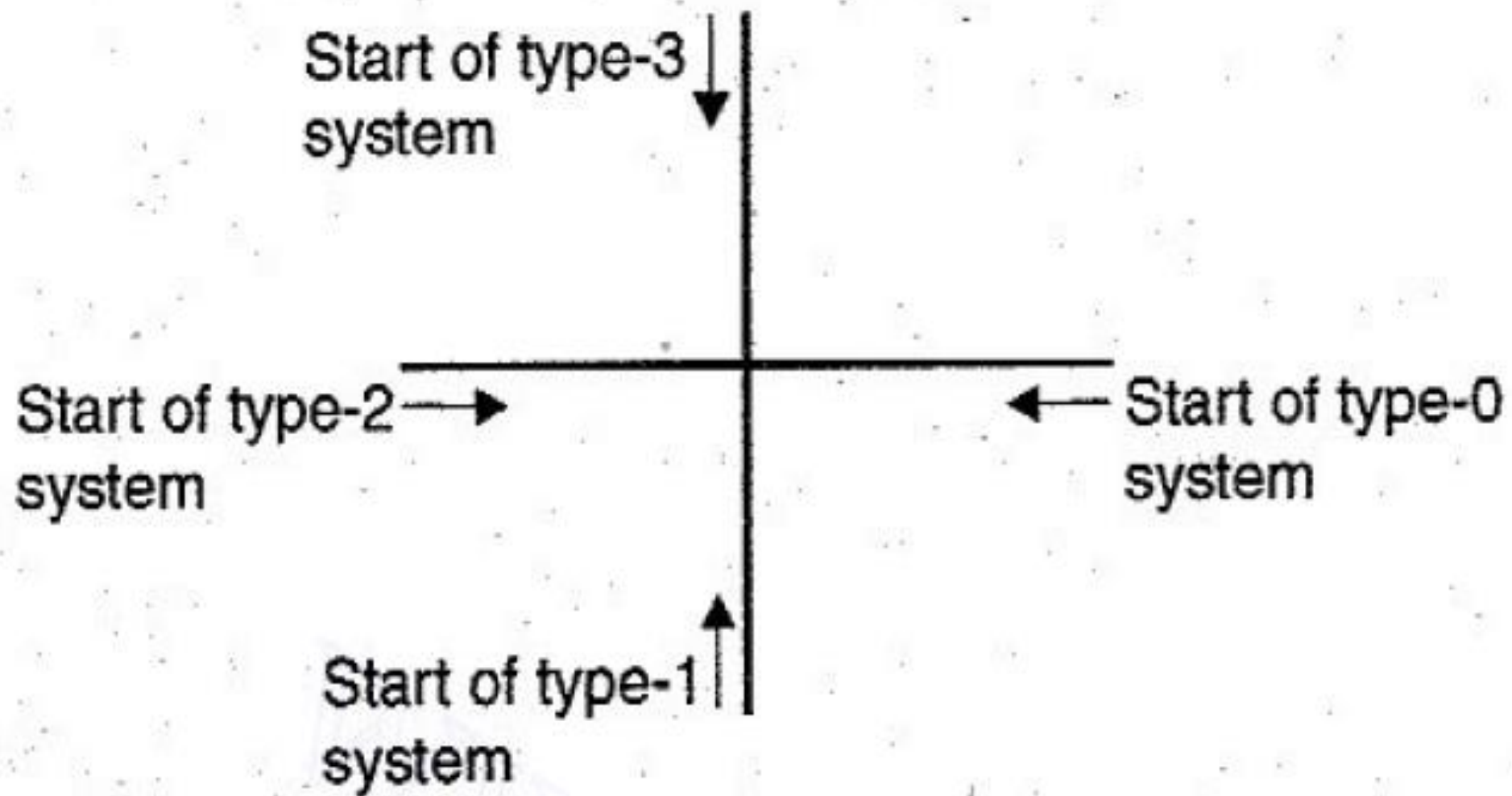
The polar plot is usually plotted on a polar graph sheet. The polar graph sheet has concentric circles and radial lines. The circles represent the magnitude and the radial lines represent the phase angles. Each point on the polar graph has a magnitude and phase angle. The

that point. In polar graph sheet a positive phase angle is measured in anticlockwise from the reference axis (0°) and a negative angle is measured clockwise from the reference axis (0°).



For minimum phase transfer function with only poles, type number of the system determines the quadrant at which the polar plot starts and the order of the system determines the quadrant at which the polar plot ends. The minimum phase systems are systems with all poles and zeros on left half of s -plane. The start and end of polar plot of all pole minimum phase system are shown in fig 3.21 & 3.22

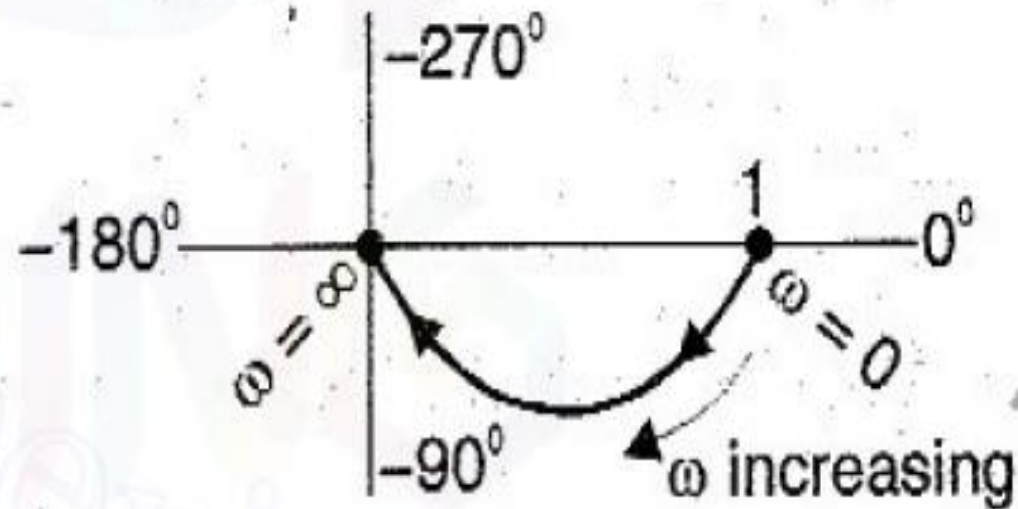




Typical Sketches of Polar Plot

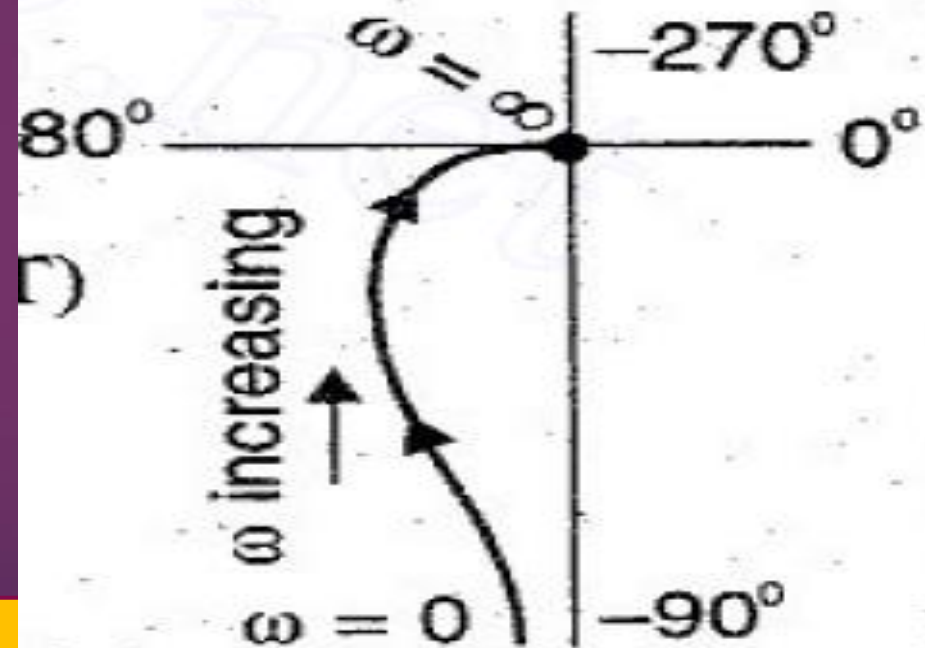
Type : 0, Order : 1

$$G(s) = \frac{1}{1+sT}$$



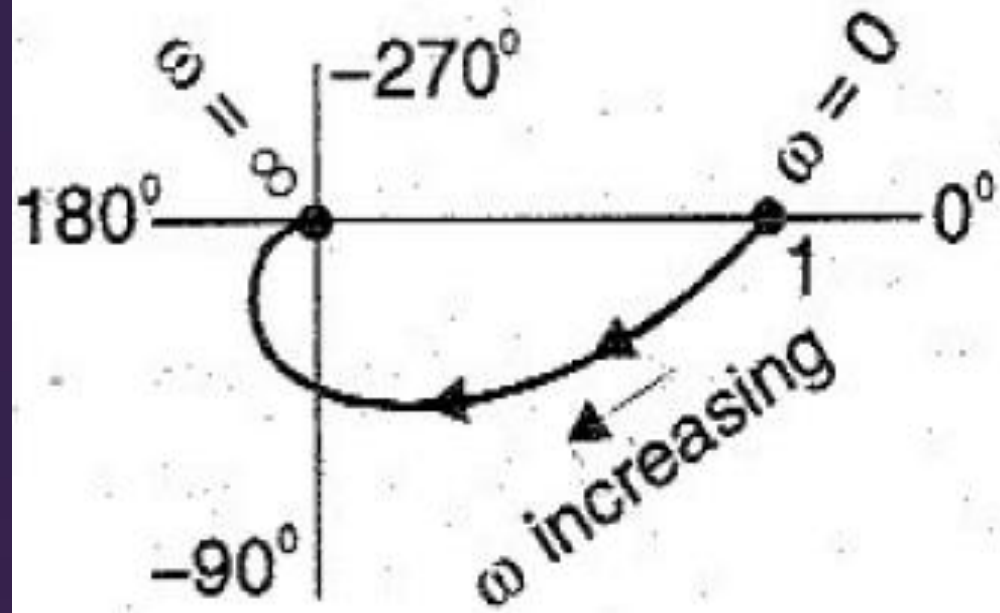
Type : 1, Order : 2

$$G(s) = \frac{1}{s(1+sT)}$$



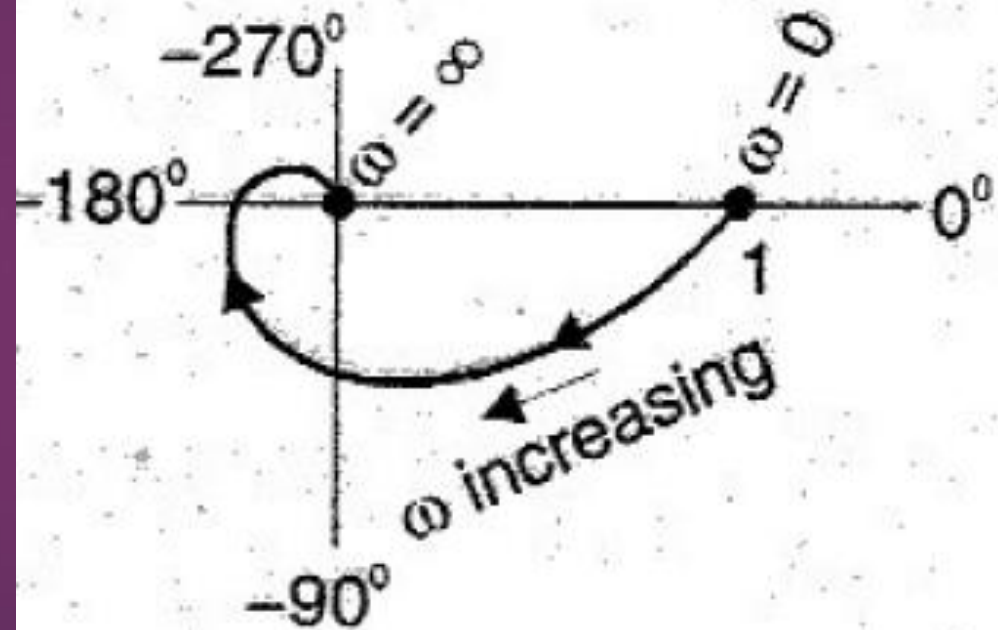
Type : 0, Order : 2

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$



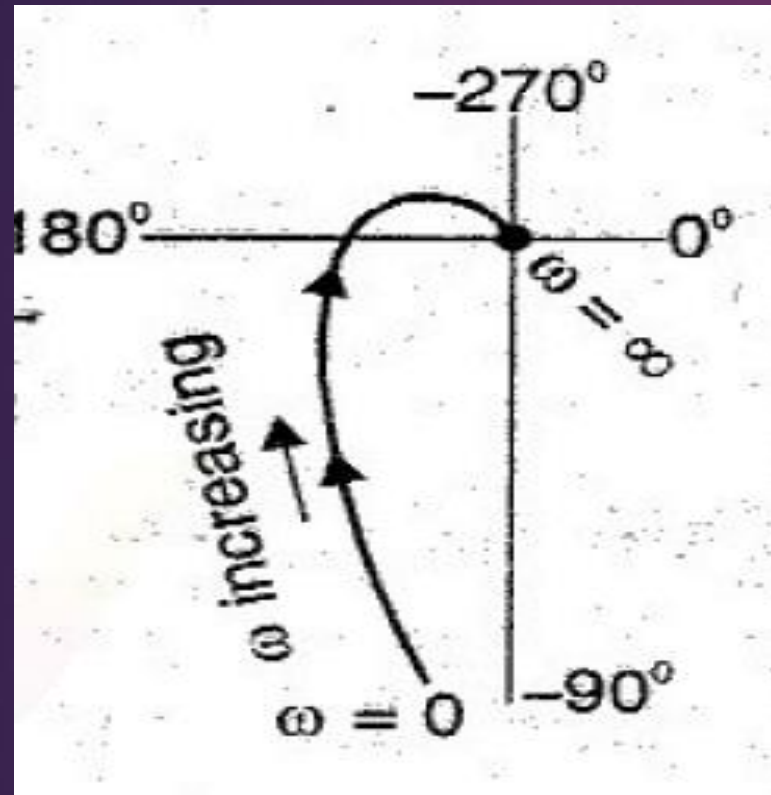
Type : 0, Order : 3

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$$



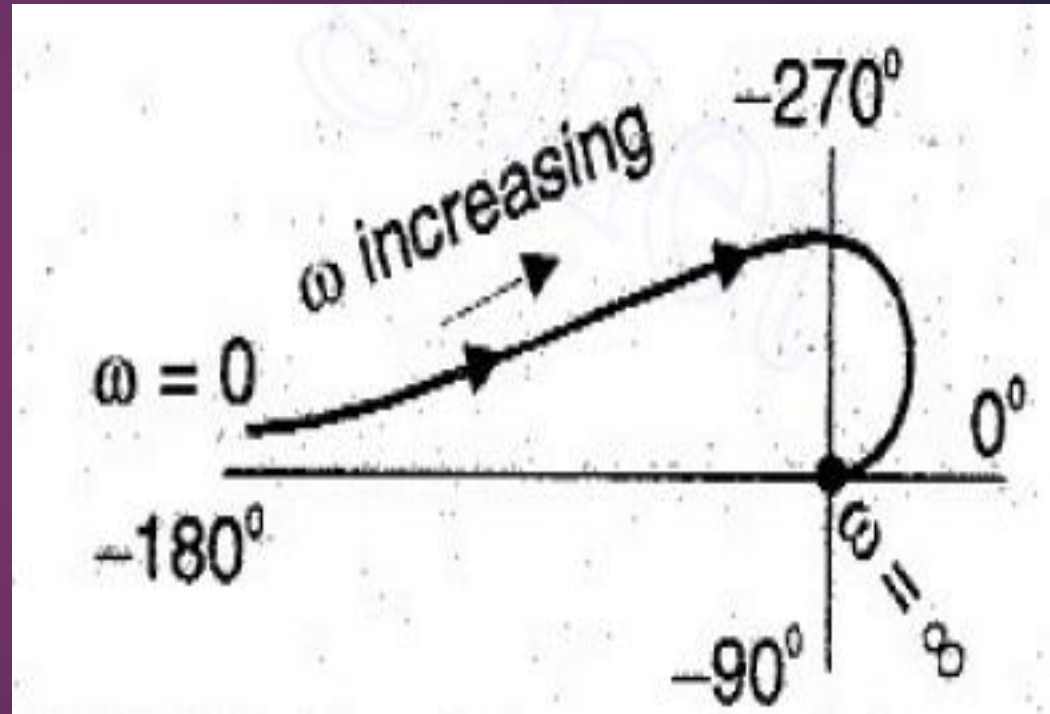
Type : 1, Order : 3

$$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$



Type : 2, Order : 4

$$G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)}$$



Example #2

The open loop transfer function of a unity feedback system is given by $G(s) = 1/s(1+s)(1+2s)$. Sketch the polar plot and determine the gain margin and phase margin.

Given that, $G(s) = 1/s(1+s)(1+2s)$

Put $s = j\omega$.

$$\therefore G(j\omega) = \frac{1}{j\omega (1+j\omega) (1+j2\omega)}$$

The corner frequencies are $\omega_{c1} = 1/2 = 0.5$ rad/sec and $\omega_{c2} = 1$ rad/sec.

$$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+j2\omega)}$$

$$= \frac{1}{\omega \sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$\therefore |G(j\omega)| = \frac{1}{\omega \sqrt{(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega \sqrt{1+4\omega^2 + \omega^2 + 4\omega^4}} = \frac{1}{\omega \sqrt{1+5\omega^2 + 4\omega^4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

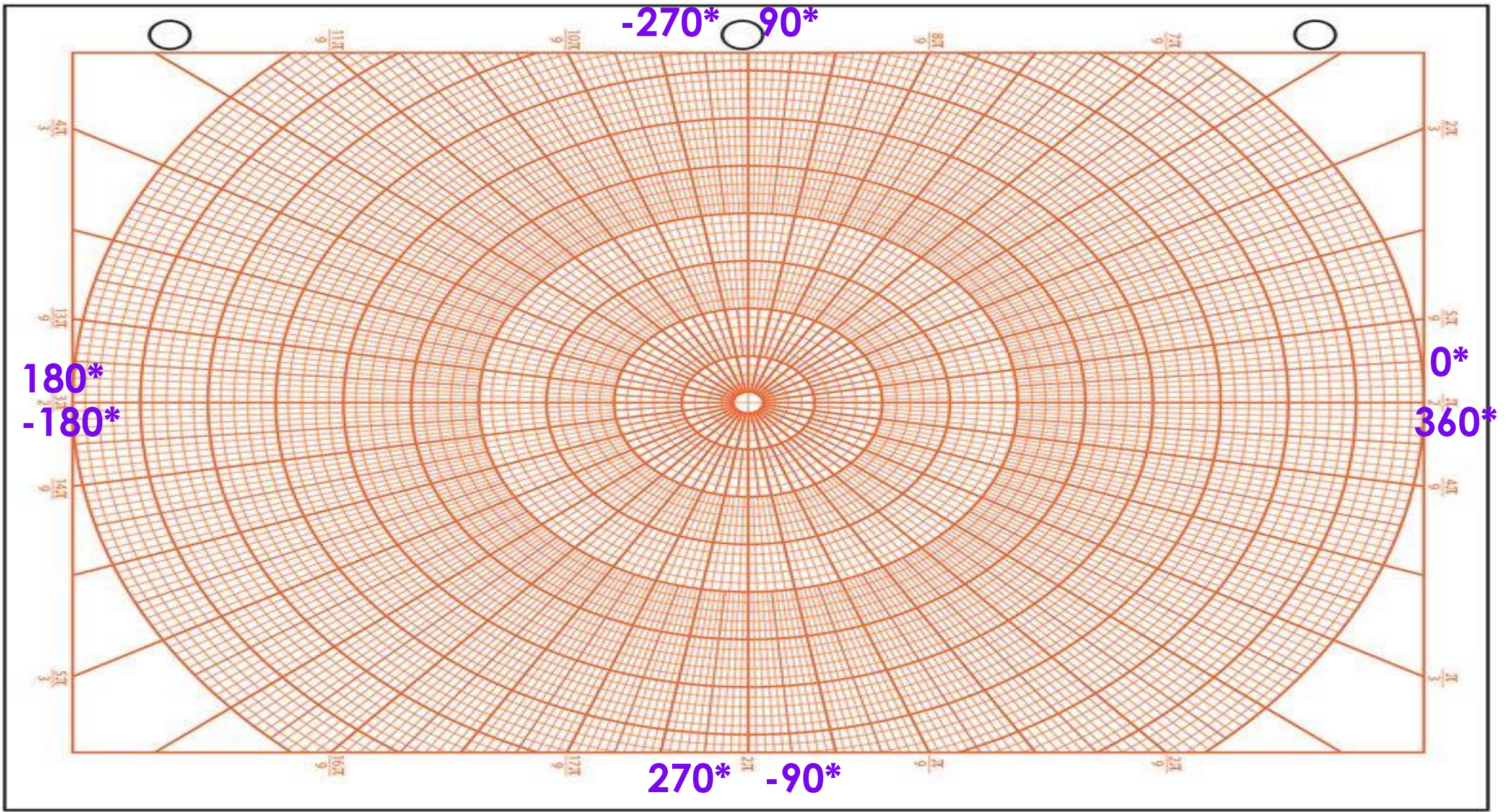
TABLE-1 : Magnitude and phase of $G(j\omega)$ at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$ G(j\omega) $	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$ deg	-144	-150	-156	-162	-171	-179.5	-198

RESULT

Gain margin, $K_g = 1.4286$

Phase margin, $\gamma = +12^\circ$



-270* **90***

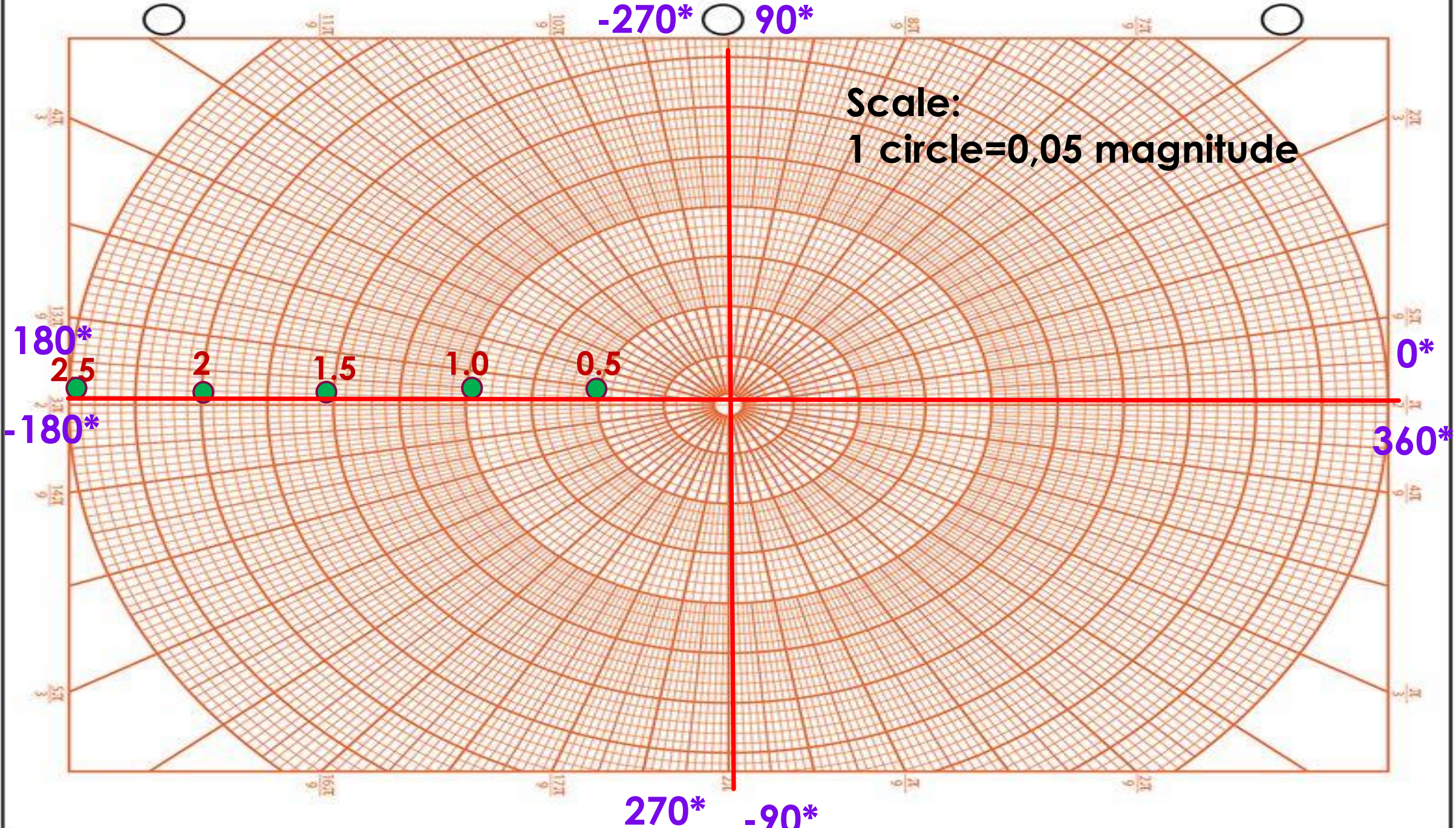
180*
-180*

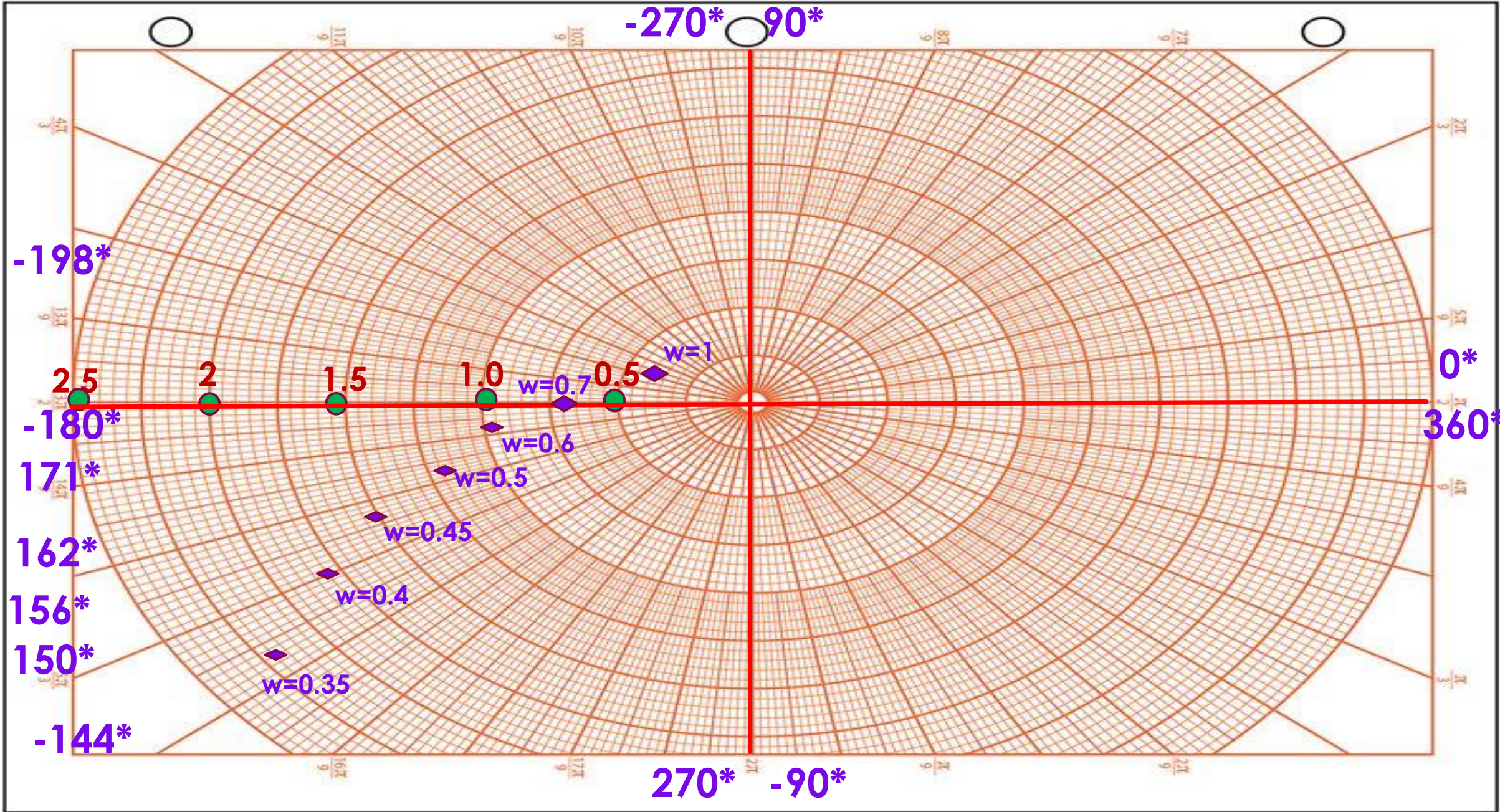
0*
360*

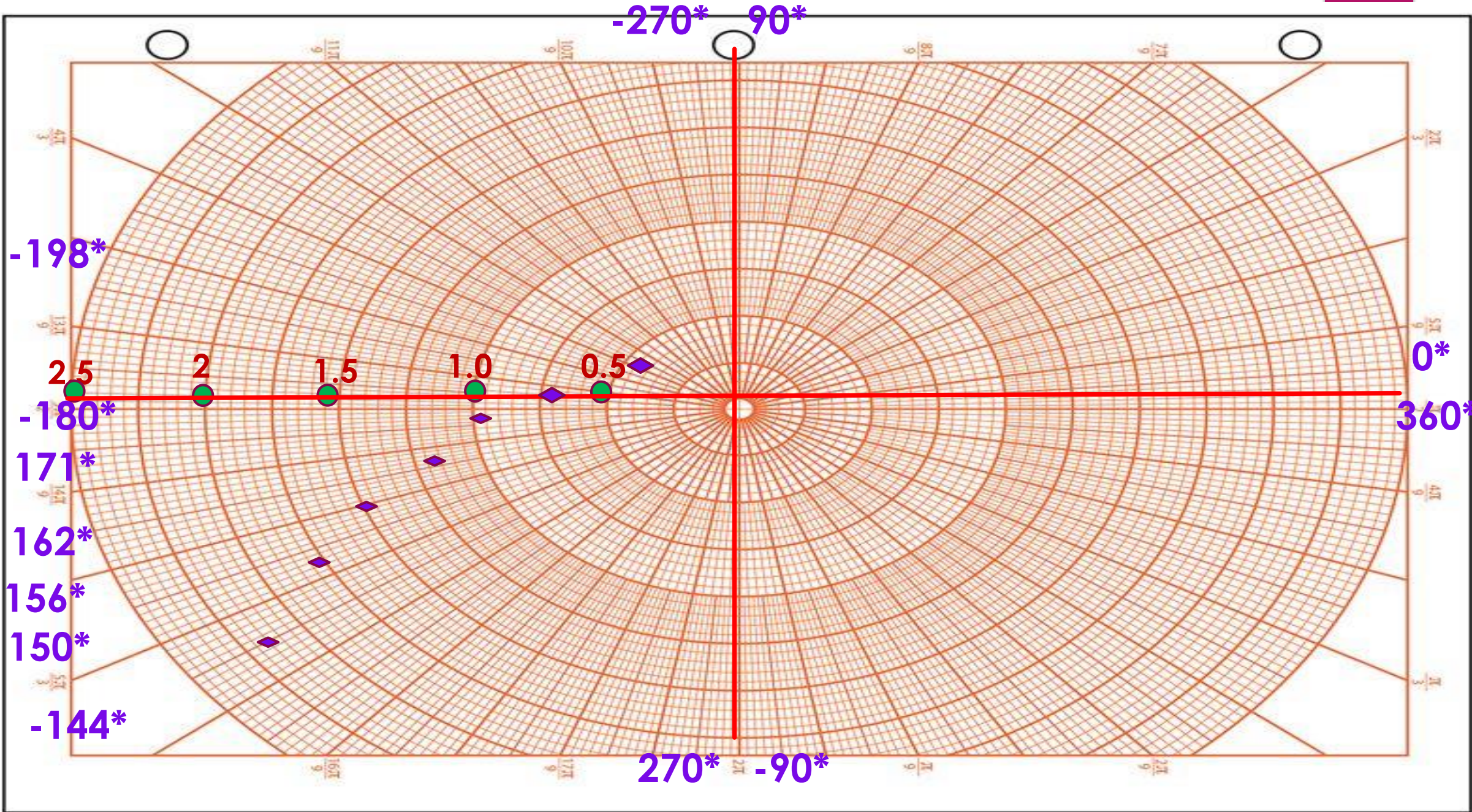
270* **-90***

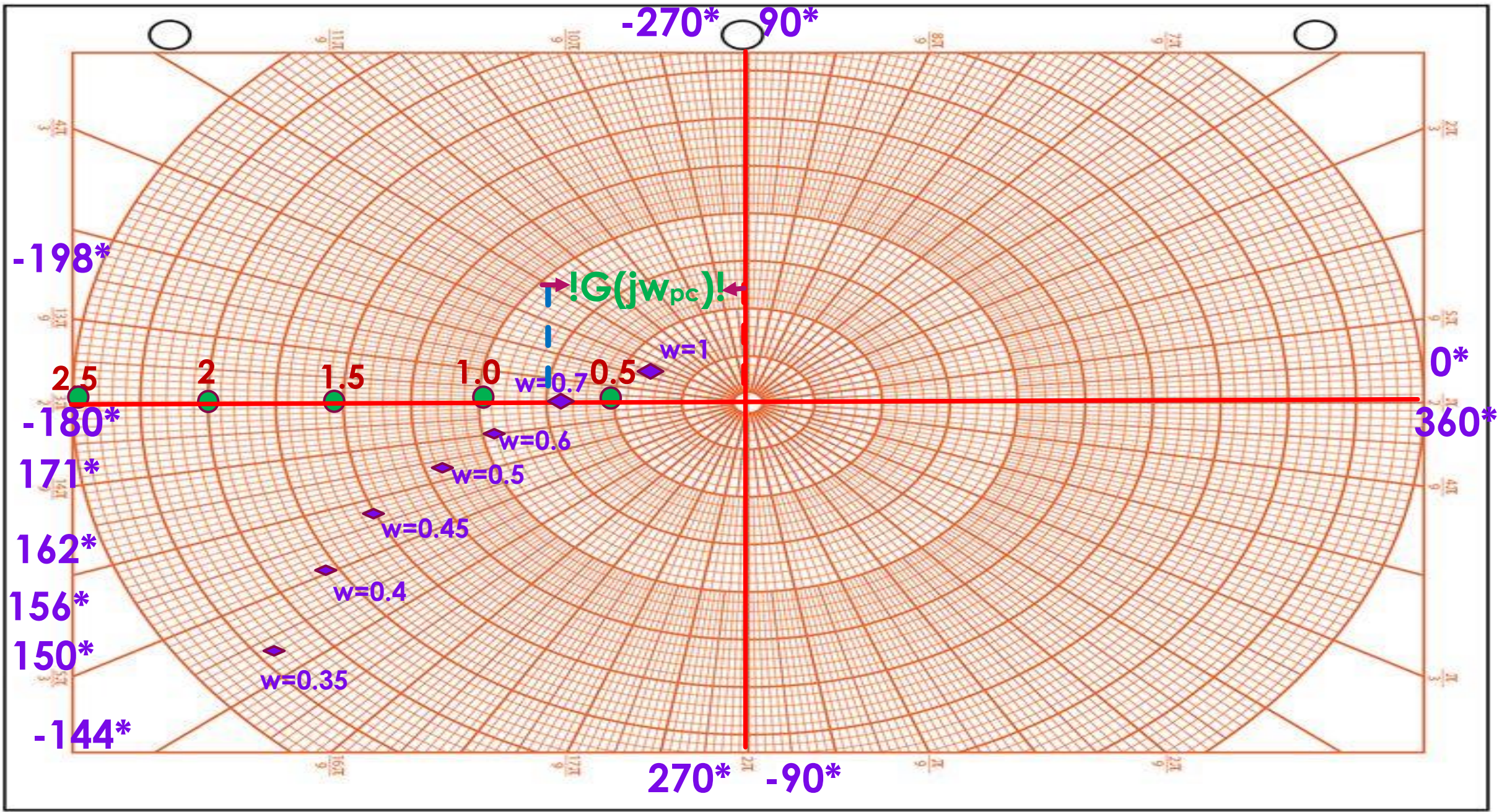
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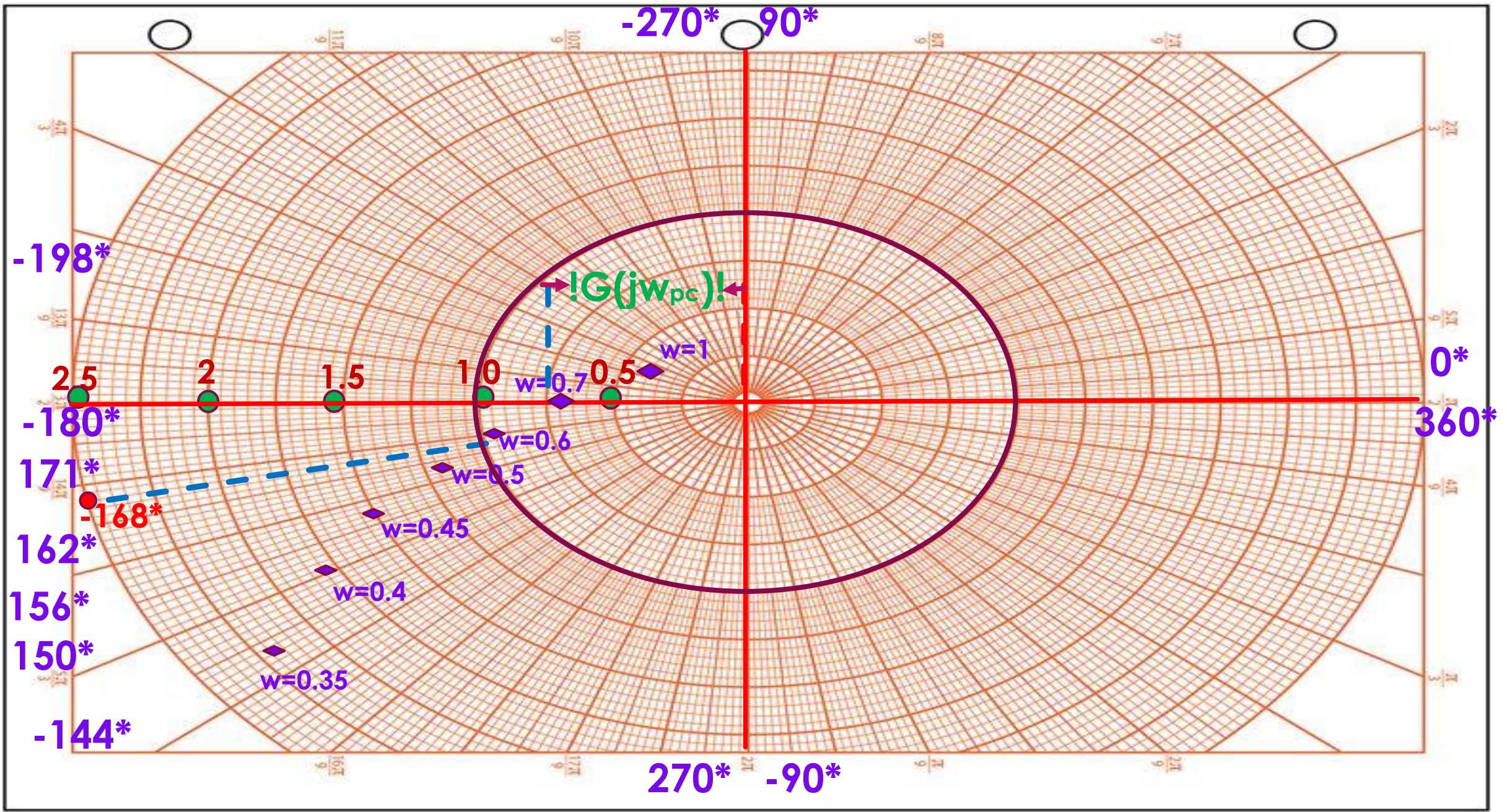
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$ G(j\omega) $	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$ deg	-144	-150	-156	-162	-171	-179.5 ≈ -180	-198

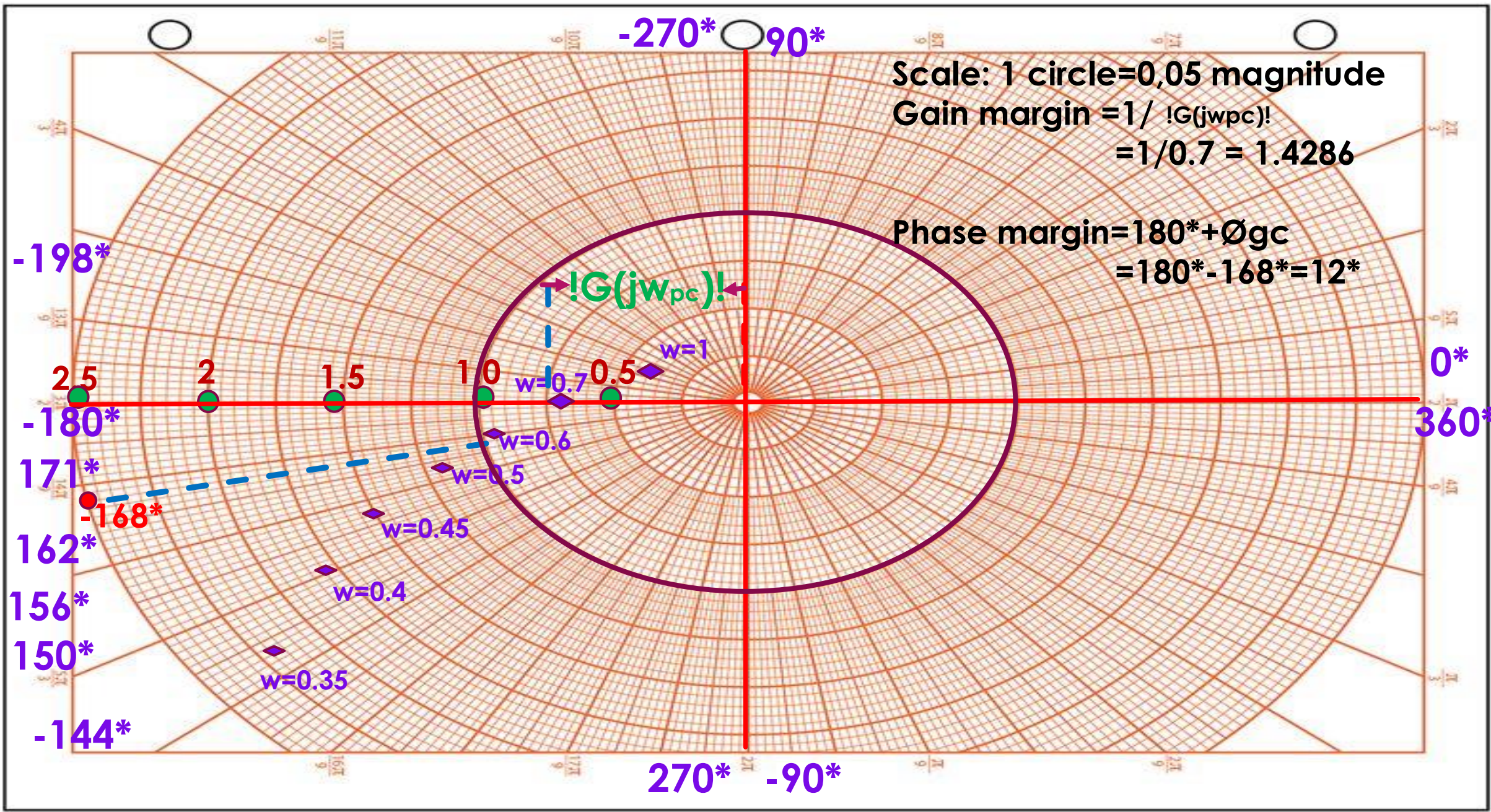












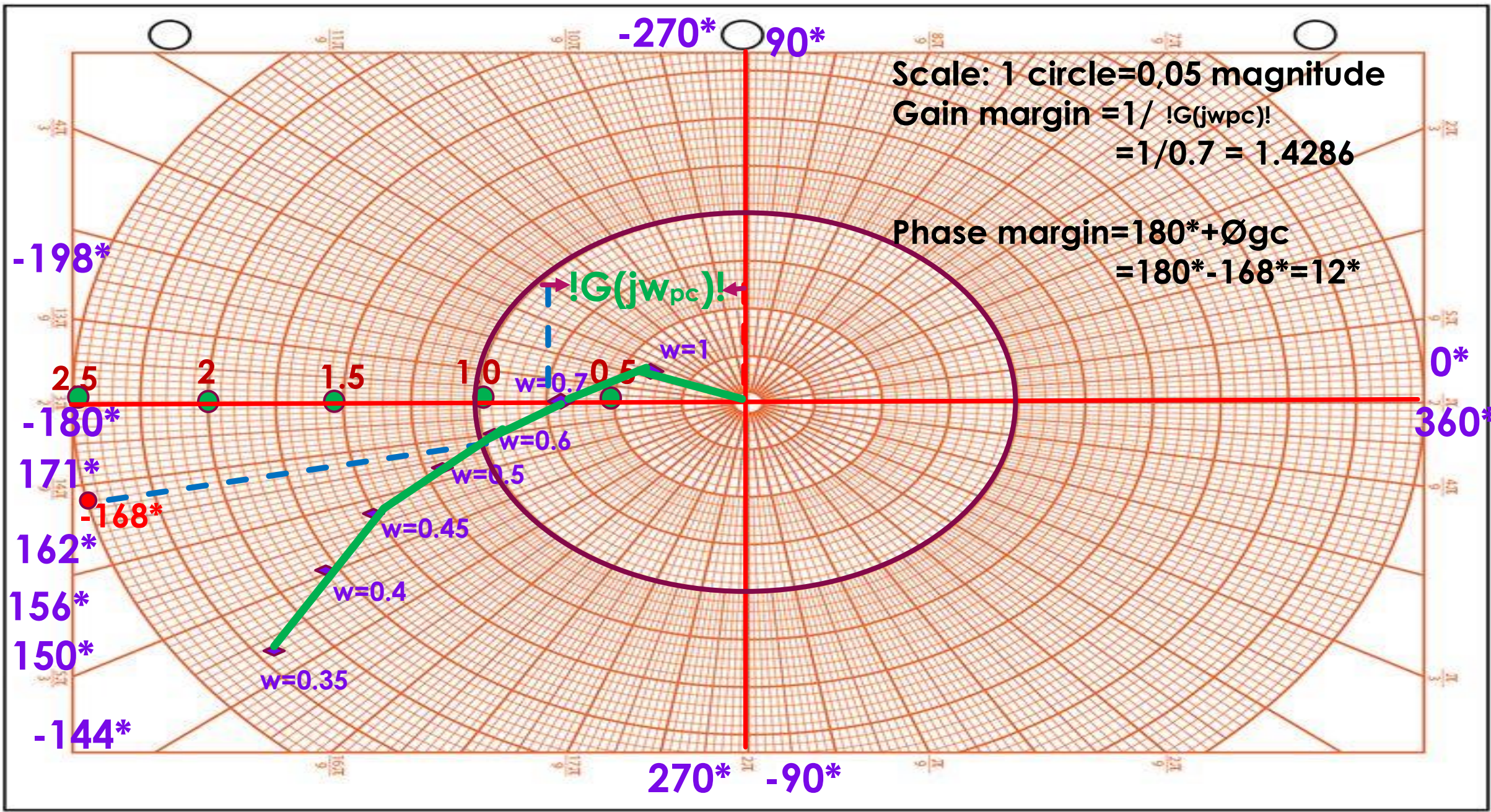
Scale: 1 circle=0,05 magnitude
 Gain margin = $1/|G(jw_{pc})|$
 $=1/0.7 = 1.4286$

Phase margin = $180^\circ + \phi_{gc}$
 $=180^\circ - 168^\circ = 12^\circ$

-198*
-180*
171*
162*
156*
150*
-144*

-270* 90*
270* -90*

0*
360*



Example #2

The open loop transfer function of a unity feedback system is given by $G(s) = 1/s^2(1+s)(1+2s)$. Sketch the polar plot and determine the gain margin and phase margin.

SOLUTION

Given that, $G(s) = 1/s^2(1+s)(1+2s)$

$$\text{Put } s = j\omega, \therefore G(j\omega) = \frac{1}{(j\omega)^2 (1+j\omega) (1+j2\omega)}$$

The corner frequencies are $\omega_{c1} = 0.5 \text{ rad/sec}$ and $\omega_{c2} = 1 \text{ rad/sec}$.

$$\text{Put } s = j\omega, \therefore G(j\omega) = \frac{1}{(j\omega)^2 (1+j\omega) (1+j2\omega)}$$

$$G(j\omega) = \frac{1}{\omega^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}} \angle(-180 - \tan^{-1}\omega - \tan^{-1}2\omega)$$

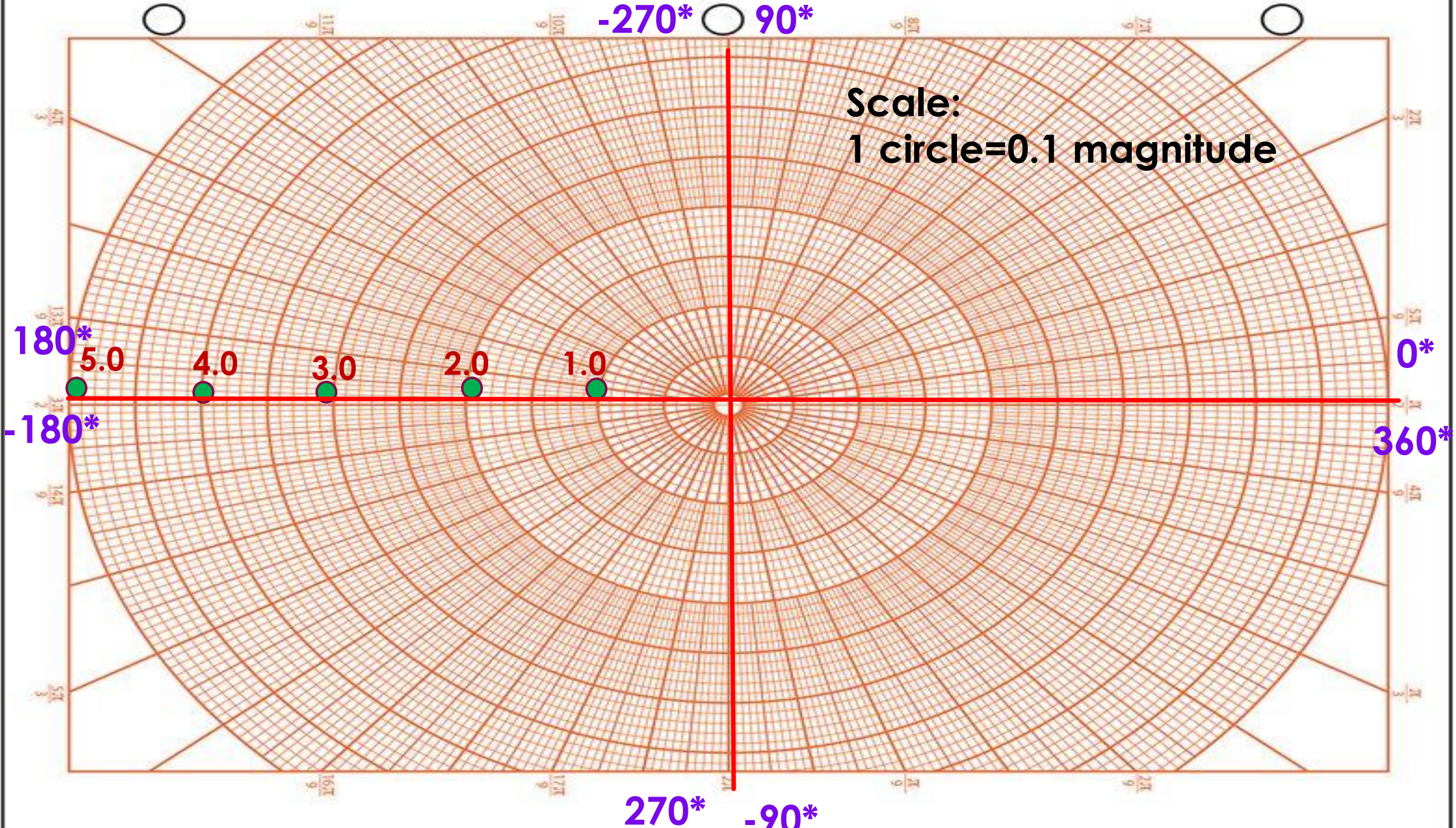
$$|G(j\omega)| = \frac{1}{\omega^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}} = \frac{1}{\omega^2 \sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$= \frac{1}{\omega^2 \sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega.$$

TABLE-1 : Magnitude and phase plot of $G(j\omega)$ at various frequencies

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$ G(j\omega) $	3.3	2.5	1.9	1.5	1.2	$0.97 \approx 1$	0.8	0.3
$\angle G(j\omega)$ deg	-246	-251	-256	-261	-265	-269	-273	-288



Scale:
1 circle=0.1 magnitude

180*

-180*

0*

360*

-270* 90*

270* -90*

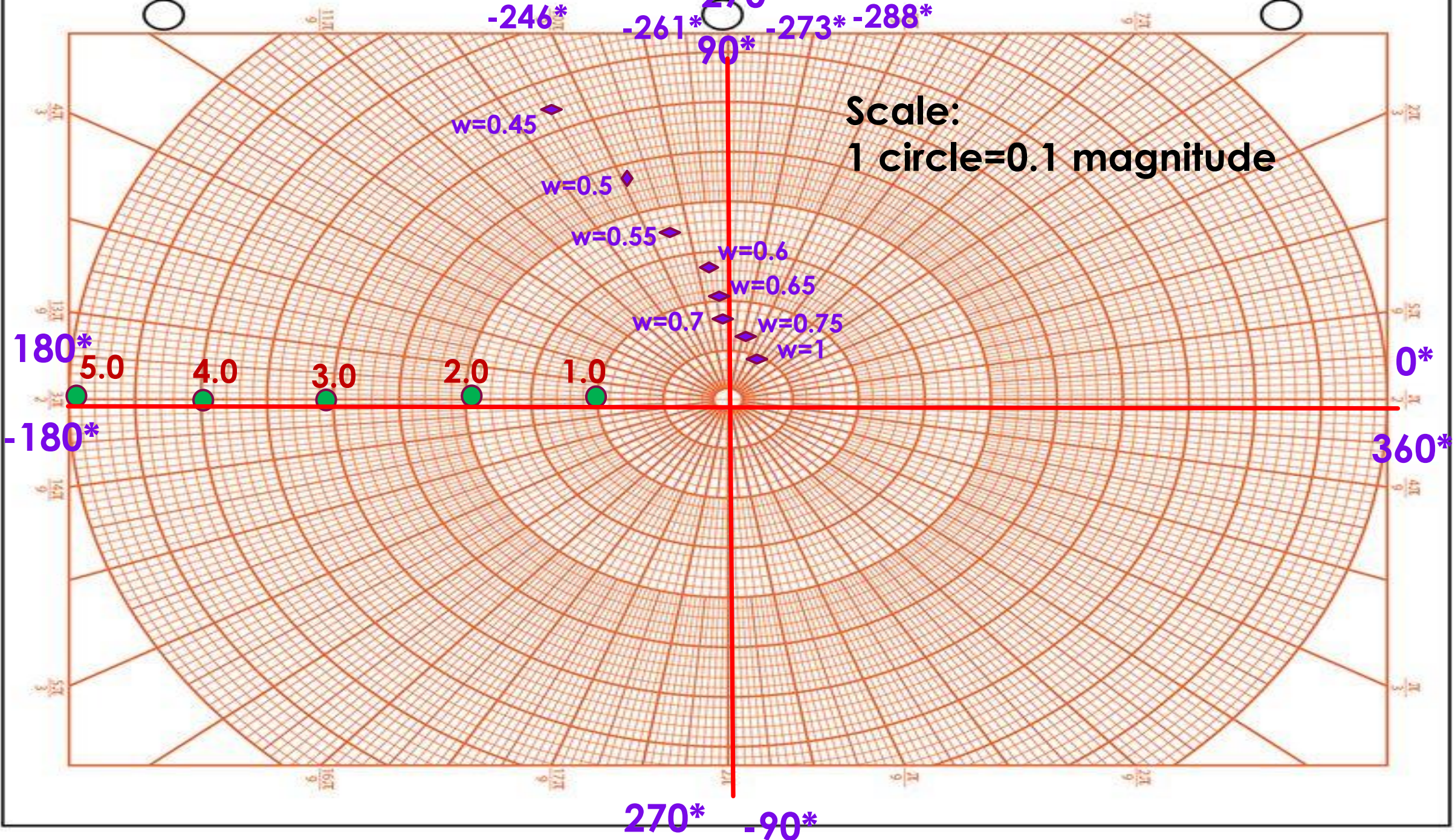
5.0

4.0

3.0

2.0

1.0



180*

-180*

0*

360*

270*

-90*

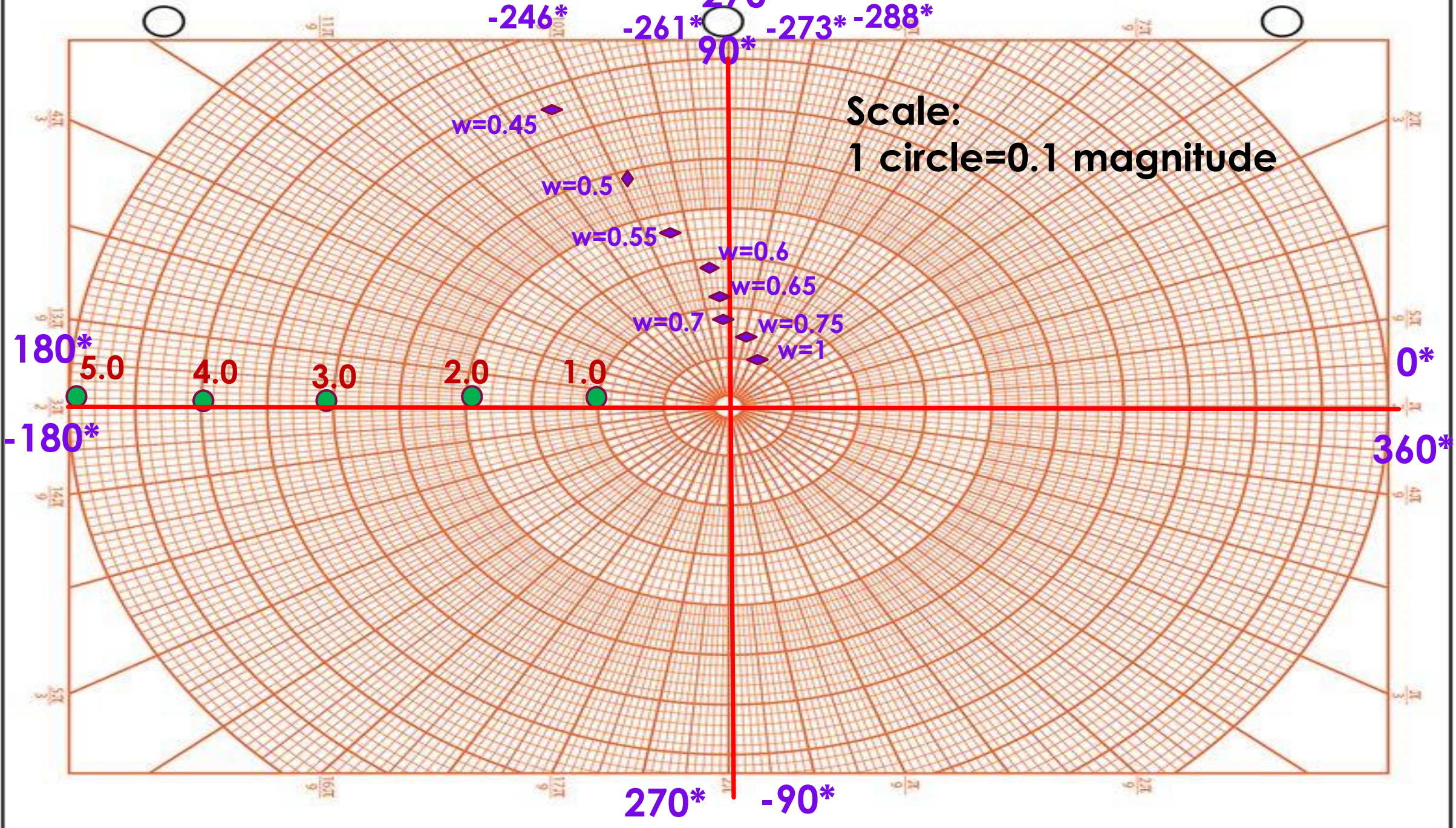
-246*

-261*

90*

-273*

-288*



Scale:
1 circle=0.1 magnitude

180*

0*

-180*

360*

270*

-90*

w=0.45

w=0.5

w=0.55

w=0.6

w=0.65

w=0.7

w=0.75

w=1

5.0

4.0

3.0

2.0

1.0

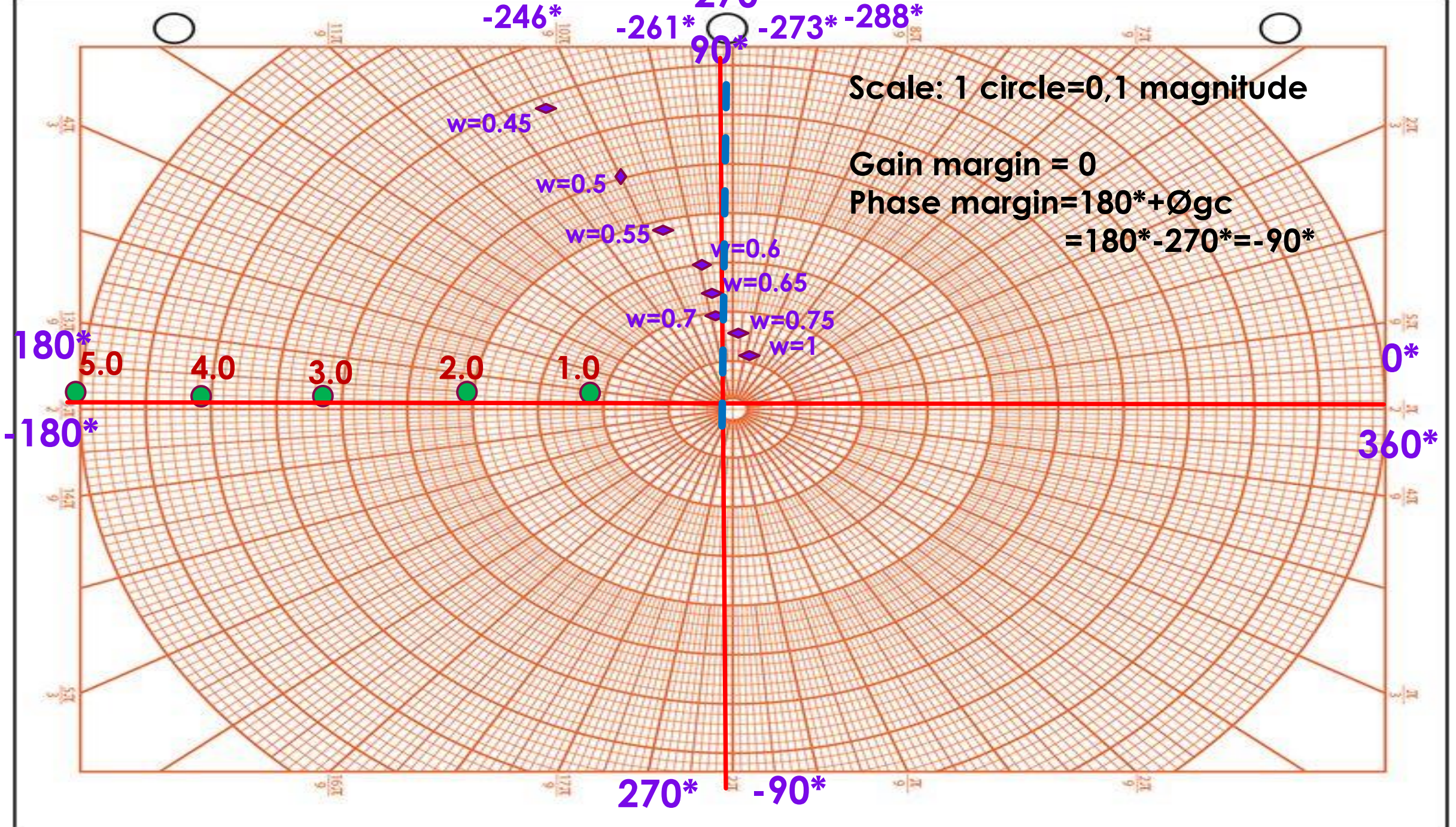
-246*

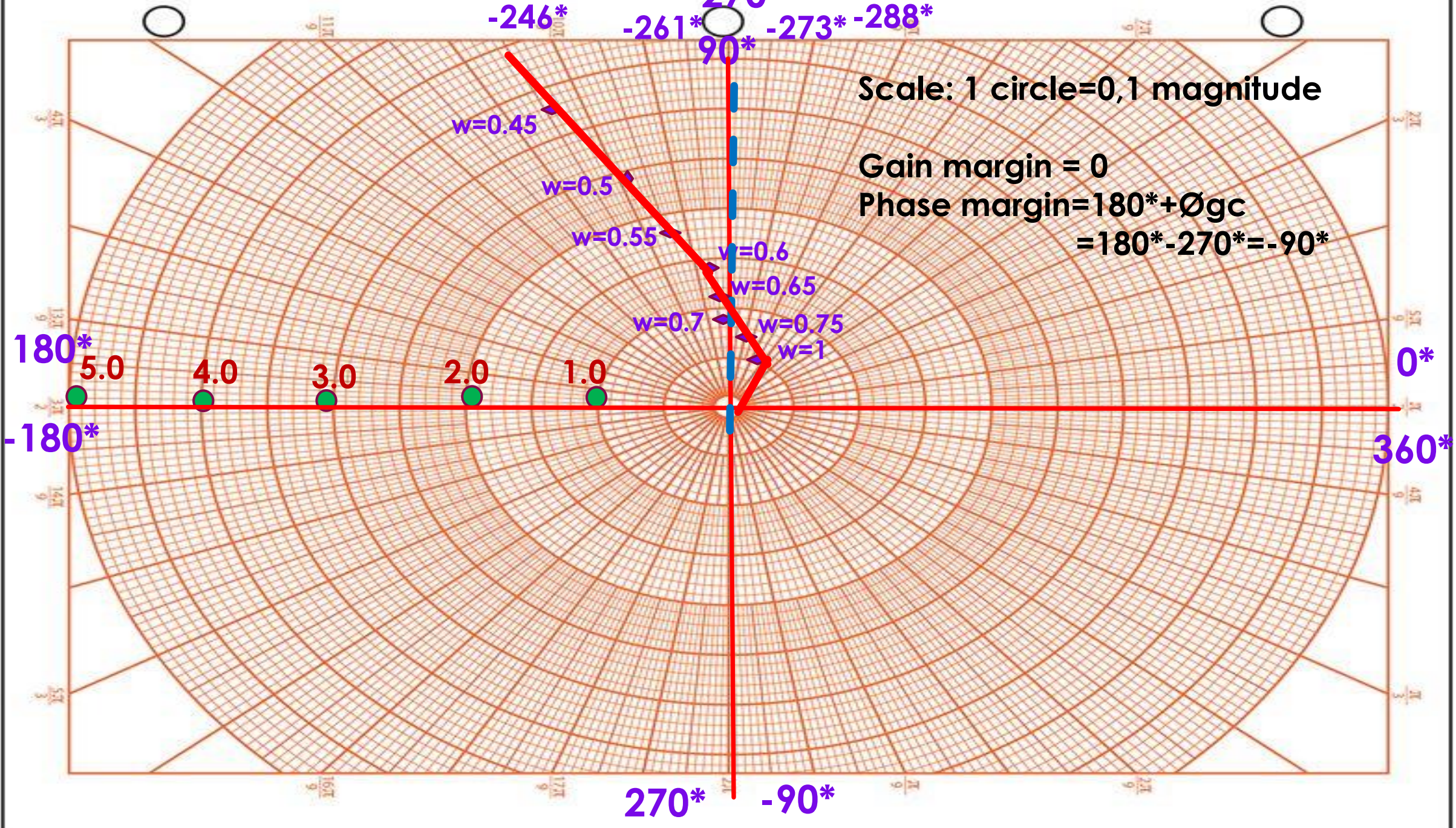
-261*

-270*

-273*

-288*





Lead, Lag, and Lead Lag Compensators

<https://www.youtube.com/watch?v=9YRhTY-W1TY>

<https://www.youtube.com/watch?v=-jevVOpeFQs>

Thank You